PHYS-3203 Homework 11 NOT TO BE HANDED IN

This homework is due to `https://uwcloud.uwinnipeg.ca/s/NwC99SeB7qHz9Ky` by 10:59PM on the due date. Your file(s) must be in PDF format; they may be black-and-white scans or photographs of hardcopies (all converted to PDF), PDF prepared by LaTeX, or PDF prepared with a word processor *using an equation editor*.

1. **Masses with Unbalanced Springs** *from a problem by Cline*

   Consider two identical masses m and three springs arranged in a line between two walls so that each mass is attached to a wall by a spring and the third spring connects the masses. The spring constants are chosen so the Lagrangian becomes

   $$L = \frac{m}{2}\left(\dot{x}_1^2 + \dot{x}_2^2\right) - \frac{k}{2}\left(4x_1^2 - 4x_1x_2 + 7x_2^2\right) \ , \tag{1}$$

   where x_1 and x_2 are the displacements of the two masses from their equilibrium positions.

   (a) Find the equations of motion (Euler-Lagrange equations).

   (b) We can write the normal modes of the system as vectors $[x_1\ x_2]^T = [B_1\ B_2]^T \exp(i\omega t)$. Write the equations of motion for a normal mode as a matrix equation.

   (c) Find the normal mode frequencies.

   (d) By substitution into the Lagrangian (1), verify that $\eta_1 = \sqrt{m/5}(x_1 - 2x_2)$ and $\eta_2 = \sqrt{m/5}(2x_1 + x_2)$ are respectively the normal coordinates for the higher and lower normal mode frequencies.