PHYS-3202 Homework 9 Due 1 Dec 2021

This homework is due to `https://uwcloud.uwinnipeg.ca/s/wxqoYpEEa8WT2LX` by 10:59PM on the due date. Your file(s) must be in PDF format; they may be black-and-white scans or photographs of hardcopies (all converted to PDF), PDF prepared by LaTeX, or PDF prepared with a word processor *using an equation editor.*

1. **Some Eigenvectors** *inspired by Riley, Hobson, & Bence and Arfken & Weber*

   Define the matrices

$$A = \begin{bmatrix} 0 & 0 & 1 \\ 0 & 1 & 0 \\ 1 & 0 & 0 \end{bmatrix} , \quad B = \begin{bmatrix} 0 & 1 & 1 \\ 1 & 0 & 1 \\ 1 & 1 & 0 \end{bmatrix} , \quad C = \begin{bmatrix} 5 & 0 & 2 \\ 0 & 1 & 0 \\ 2 & 0 & 2 \end{bmatrix} . \tag{1}$$

   (a) By matrix multiplication, show that the vectors $\vec{x} = \begin{bmatrix} 1 & 0 & -1 \end{bmatrix}^T$, $\vec{y} = \begin{bmatrix} 1 & 1 & 1 \end{bmatrix}^T$, and $\vec{z} = \begin{bmatrix} 1 & -2 & 1 \end{bmatrix}^T$ are eigenvectors of both A and B and find their eigenvalues for each matrix.

   (b) Find an orthonormal basis of eigenvectors for C. *Hint:* If you choose carefully, you can find the eigenvectors to be orthogonal from the start. Otherwise, you may need to use the Gram-Schmidt process to orthonormalize your initial choice (you can find the Gram Schmidt process in a mathematical physics textbook if necessary).

2. **Some Moments of Inertia**

   Calculate the moment of inertia of each of the following objects through the specified axis.

   (a) A solid sphere with total mass M and radius R and uniform density about any axis through its center of mass.

   (b) A uniform circular disk of mass M, radius R, and negligible thickness about the axis perpendicular to the disk through the center of mass.

3. **Finding Principal Axes**

   Four identical small balls of mass m each are at the following locations in the xy plane: $(x, y, z) = (a, 0, 0), (-a, 0, 0), (a/\sqrt{3}, 2a/\sqrt{3}, 0), (-a/\sqrt{3}, -2a/\sqrt{3}, 0)$. They are held together by very light rods. Treat the balls as idealized point particles and the rods as massless.

   (a) Find the (3D) inertia tensor of this object around the origin.

   (b) Find the principal axes and corresponding moments of inertia for the object.