PHYS-3202 Homework 7 Due 17 Nov 2021

This homework is due to `https://uwcloud.uwinnipeg.ca/s/wxqoYpEEa8WT2LX` by 10:59PM on the due date. Your file(s) must be in PDF format; they may be black-and-white scans or photographs of hardcopies (all converted to PDF), PDF prepared by LaTeX, or PDF prepared with a word processor *using an equation editor*.

1. **Pendulum in a Bus**

   A pendulum of length l hangs from the ceiling of a bus, which is waiting at a red stoplight (stationary with respect to the earth). At time $t = 0$, the light turns green, and the bus starts driving with constant horizontal acceleration $\vec{a}$. Describe the pendulum's position by θ, the angle from the downward vertical (parallel to the Earth's gravitational acceleration $\vec{g}$).

   (a) Find the equilibrium position θ_0 of the pendulum when the bus is accelerating. Describe how you find your answer from the perspective of both an inertial observer on the ground and an accelerating observer on the bus.

   (b) Describe the motion of the pendulum if it is hanging straight down at the time the bus starts accelerating. Assume $|\vec{a}| \ll g$. *Hint:* start by writing Newton's second law in the θ direction in the accelerating frame, then show it is approximately the same as for the harmonic oscillator.

2. **Puck on Turntable** *inspired by Taylor*

   Consider a puck of mass m on a turntable with angular velocity $\vec{\omega} = \omega\hat{k}$, ie, counterclockwise rotation. Work in a rotating frame with origin at the center of the turntable and x, y axes along set lines on the turntable.

   (a) Write Newton's second law for the x and y coordinates. If we define $u = x + iy$, what is the equation of motion for u? Assume that there is no friction and that ω is constant.

   (b) Solve for $x(t)$ and $y(t)$ given the initial conditions $\vec{r}(0) = x_0\hat{x}$ and $\dot{\vec{r}}(0) = v_x\hat{x} + v_y\hat{y}$. Sketch your answer for $x_0 = 0, v_y = 0$. *Hint:* Guess a complex exponential solution for the variable u from the previous part. Then show that there is a double root for the frequency, like a critically damped harmonic oscillator.

   (c) Explain qualitatively why you might prefer to work in this rotating reference frame as opposed to an inertial frame if the puck and turntable experience kinetic friction opposite $\vec{v}$ or a drag force $-2m\gamma\vec{v}$, where $\vec{v}$ is the relative velocity of the puck and turntable.

3. **Rotating Cylindrical Space Vessel** *inspired by KB 5.9*

   One possible way residents of a space ship or space station can experience "artificial gravity" is if the vessel rotates around some axis. Consider a cylindrical vessel of radius R rotating with angular frequency ω around the cylinder axis.

   (a) The centrifugal force provides an effective gravitational force for stationary objects. At what height h above the "ground" at radius R a second story in a building have to be in order to have only 90% of the gravitational acceleration?

   (b) A train runs around the circumference of the vessel in the direction of the vessel's rotation with speed v relative to the ground. What is the increase in the effective weight (as measured by the normal force) for objects on the train compared to their weight at rest on the ground? Explain your answer first from the rotating frame of the vessel and then

from the rotating frame in which the train is at rest. *Hint:* in the frame of the ground, the objects on the train have a centripetal acceleration.