PHYS-3202 Homework 4 Due 20 Oct 2021

This homework is due to `https://uwcloud.uwinnipeg.ca/s/wxqoYpEEa8WT2LX` by 10:59PM on the due date. Your file(s) must be in PDF format; they may be black-and-white scans or photographs of hardcopies (all converted to PDF), PDF prepared by LaTeX, or PDF prepared with a word processor *using an equation editor.*

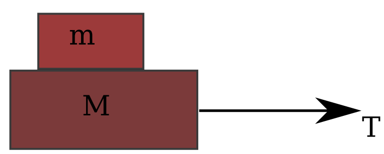
1. **Multiple-Choice from a Previous Exam** *some from MIT OpenCourseWare*

   For each part, choose the correct answer from the options given and explain your answer in no more than two lines.

   (a) A triathlete rides a bike up a hill. In what direction does the friction from the ground act on the bike? The wheels of the bike do not slip on the ground.
   A. $F_{friction} = 0$ B. down the hill C. up the hill D. perpendicular to the ground

   (b) A hockey puck of mass m and velocity $\vec{v}$ strikes a stationary puck of mass $3m$ elastically. Which of the following could be the final velocities of the two pucks?
   A. $-\vec{v}/2$ and $\vec{v}/2$ B. $\vec{v}/2$ and $3\vec{v}/2$ C. $-3\vec{v}$ and $\vec{v}$ D. 0 and $\vec{v}/3$

   (c) A block of mass m sits on top of a larger block of mass M as in the figure. The lower block is pulled so that both blocks have acceleration a to the right; the upper block does not move with respect to the lower one.

   The coefficients of static and kinetic friction *between the blocks* are μ_s and μ_k respectively. What are the magnitude and direction of the force of friction acting on the upper block?
   A. $\mu_k mg$ to the left B. $\mu_s mg$ to the left C. $\mu_s mg$ to the right D. ma to the right
   E. $(M + m)a$ to the right

   (d) Two identical particles moving in a central force have concentric circular orbits of radii r_1 and $r_2 = 2r_1$ respectively. The angular momenta of the two particles are J_1 and $J_2 = 3J_1$, and their periods are T_1 and T_2 respectively. What is the ratio T_2/T_1?
   A. 4 B. 4/3 C. 1/2 D. 1/3

2. **Isotropic Oscillator with Damping**

   Consider a three-dimensional isotropic harmonic oscillator with spring constant $k = m\omega_0^2$ that also experiences an isotropic linear damping force $\vec{F}_{damp} = -2m\gamma\vec{v}$. Assume $\gamma \ll \omega_0$.

   (a) The damping force also provides a torque on the oscillator. Show that this torque is always proportional to the angular momentum. Use this fact to show that the angular momentum has exponential time dependence and further that motion remains in a plane.

   (b) Find the general solution for the x position of the oscillator as a real function of time (ie, in a form that is manifestly a real number). Note that the general solution for y is the same but with different integration constants due to different initial conditions.

   (c) Use your general solution from part (b) to find the position as a function of time given initial conditions $\vec{r}(0) = x_0\hat{\imath}$ and $\vec{v}(0) = v_0\hat{\jmath}$. *Hint:* Be careful about time derivatives.

(d) Find the angular momentum of your solution from part (c) and confirm that it has the correct exponential time dependence that you found in part (a).

3. **An Extended Isotropic Spring** *from Kibble & Berkshire*

   One end of a spring is attached to the origin, and the other is attached to a mass m. The spring can swivel freely in two dimensions (horizontal, so we neglect gravity). The equilibrium length of the spring is a, so the restoring force on the mass is $-k(\rho - a)$ in cylindrical coordinates. Ignore damping. Note that cylindrical coordinates in the xy plane are the same as spherical polar coordinates in the equatorial plane with some renaming.

   (a) Suppose the mass oscillates linearly (ie, in the x direction with $y = 0$ at all times). What is the angular frequency ω_0 of oscillation?

   (b) For motion with angular momentum $\vec{J} = J\hat{z}$, find the effective potential for radial motion.

   (c) Suppose the mass has initial conditions $\rho = a, \varphi = 0$ and $\dot{\rho} = 0, \dot{\varphi} = \omega$. If the mass reaches a maximum radius of $2a$, find ω and the value of $\dot{\varphi}$ when $\rho = 2a$. Write your answers as multiples of the natural frequency ω_0.

   (d) Now suppose that the mass is moving in a circular orbit of radius $\rho = 2a$. Find the angular velocity of the orbit (which is also the frequency of the orbit) in terms of the natural oscillator frequency ω_0. *Hint:* Where is the circular orbit in terms of the effective potential for radial motion?

4. **Damped Pendulum**

   An idealized pendulum consists of a mass m at the end of a massless rod of length L hanging from a pivot point at the other end. The mass is free to move on a circle of radius L around the pivot; the position is measured by the angle θ from the downward vertical. Find the angular momentum of the pendulum bob and the torque due to gravity. Assuming that there is also a damping torque $\tau_d = -2mL^2\gamma\dot{\theta}$, write the equation of motion for the pendulum from the angular momentum equation. Show that it becomes the same as the harmonic oscillator equation for small θ. Finally, find the kinetic energy in terms of $\dot{\theta}$.