PHYS-3202 Homework 2 Due 29 Sept 2021

This homework is due to `https://uwcloud.uwinnipeg.ca/s/wxqoYpEEa8WT2LX` by 10:59PM on the due date. Your file(s) must be in PDF format; they may be black-and-white scans or photographs of hardcopies (all converted to PDF), PDF prepared by LaTeX, or PDF prepared with a word processor *using an equation editor.*

1. **Hanging Spring**

   Consider a mass m on a spring with potential energy $kx^2/2$, where $x = 0$ is the equilibrium extension. Suppose the spring is hung from the ceiling (with x increasing downwards).

   (a) Write the potential energy as a function of x with the inclusion of gravity and find the new equilibrium point x_0.

   (b) Rewrite the potential in terms of $y = x - x_0$. From the form of the potential only, argue that the motion of the hanging spring is harmonic oscillation around $y = 0$ and find the frequency of oscillation. Do not solve any differential equations.

   (c) Suppose the spring is initially displaced with $y(0) = y_0$ and $\dot{y}(0) = v_0$. Find the motion $y(t)$ as a function of time in the form $a\cos(\omega t - \theta)$. What are a, ω, and θ?

2. **Work Done on a Forced Oscillator** *similar to Cline 3.5*

   Consider a harmonic oscillator with damping γ and natural frequency ω_0 that experiences a force $F(t) = F\cos(\omega t)$.

   (a) The motion of the harmonic oscillator $x(t)$ can be written as a particular solution for the force $F(t)$ plus *transient* solutions for the same harmonic oscillator with $F = 0$. Write the transient solutions for this oscillator assuming $\omega_0 > \gamma$. Describe the behavior of these solutions at late times.

   (b) In class, we found the particular solution for x when the driving force is a complex exponential $F(t) = F\exp(i\omega t)$. If that complex solution is $x_1(t)$, show that $x(t) = (x_1(t)+x_1^*(t))/2$ is a solution for the cosine driving force in this problem. What is the particular solution $x(t)$ for the cosine driving force?

   (c) The power, or work done on an object per unit time, is $P = F\dot{x}$. Find the power on the oscillator by the force $F(t)$ at time t for the steady-state solution (ie, late time solution neglecting transients). Then find its average over one period.

   (d) The damping force is $-2m\gamma\dot{x}$. Find the power by the damping force at time t and the average power over one period for the steady-state solution. Show that your result is the opposite of the power from the driving force, so the total work done on the oscillator over a period is zero.