PHYS-3202 Homework 11 NOT DUE

This homework is for your study purposes only and is not to be turned in. The second question is a bit more involved than you should expect on the exam but illustrates the concepts.

1. **Rotating Lamina**

   A rigid lamina (planar object) has principal moments I_1, I_2, and $I_3 = I_1 + I_2$, as we saw on a previous assignment. The components of the angular velocity along the corresponding principal axes are ω_1, ω_2, and ω_3 respectively. Show that $\omega_1^2 + \omega_2^2$ is constant. *Note:* you cannot assume that ω_3 is constant.

2. **Air Friction** *from Fowles & Cassiday*

   Consider a disc-shaped symmetric object with $I_1 = I_2 \equiv I < I_3$, which is appropriate for a flat cylinder. While spinning in the air, it experiences a drag-like torque $\vec{\tau} = -k\vec{\omega}$ known as air friction. The disc initially has angular velocity with $\omega_3 \gg \omega_1, \omega_2$ in terms of the components along the principal axes. *Hint:* this type of torque is one that can be analyzed easily using Euler's equations.

   (a) Show that the angular velocity around the symmetry axis $\hat{e}_3$ decreases exponentially in time.

   (b) Next, show that the angle between $\vec{\omega}$ and the symmetry axis decreases in time. That is, $(\omega_1^2 + \omega_2^2)^{1/2}$ decreases more rapidly in time than ω_3.

   (c) Finally, argue that $\vec{\omega}$ goes to a fixed angle in the $\hat{e}_1, \hat{e}_2$ plane as $t \to \infty$. You may use your solution to the previous part.