PHYS-3202 Homework 10 Due 8 Dec 2021

This homework is due to `https://uwcloud.uwinnipeg.ca/s/wxqoYpEEa8WT2LX` by 10:59PM on the due date. Your file(s) must be in PDF format; they may be black-and-white scans or photographs of hardcopies (all converted to PDF), PDF prepared by LaTeX, or PDF prepared with a word processor *using an equation editor.*

1. **Kater's reversible pendulum** *based on a problem from Thornton & Marion and others*

   Consider an object hung from a pivot a distance L from the center of mass, which is a physical pendulum with frequency $\omega = \sqrt{MgL/I}$, where I is the moment of inertia around the pivot, as we've seen. Suppose we can flip the pendulum over and hang it from a pivot a distance L' on the other side of the center of mass (with a parallel axis of rotation). If the pendulum has the same frequency of oscillation around this second pivot, show that $MLL' = I^{CM}$, the moment of inertia for the parallel axis through the center of mass. *Hint:* use the parallel axis theorem for a single axis.

2. **Inertia Tensor of a Thin Plate**

   Consider a thin plate of material, which is effectively two-dimensional (this is known as a *lamina*). In this problem, assume that the lamina lies in the $z = 0$ plane.

   (a) *from Taylor* Prove that the moments of inertia $I_{zz} = I_{xx} + I_{yy}$ and the products of inertia $I_{xz} = I_{yz} = 0$.

   In the rest of the problem, assume that the lamina is a rectangle of sides with length $2a$ parallel to the x axis and length a parallel to the y axis. The center of the rectangle is at the origin. The lamina has uniform mass surface density and total mass M. *based on questions from FC and KB*

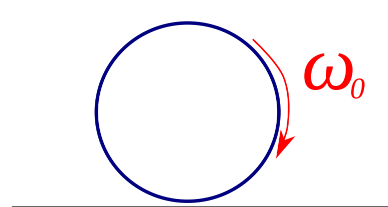
   (b) Calculate the inertia tensor around this origin given these axes. You may use the results of part (a) to simplify your calculations. (This inertia tensor applies to rotation around any axis through the center of the lamina.) What are the principal axes?

   (c) Use the parallel axis theorem to find the inertia tensor of the lamina around the corner located at $x = -a, y = -a/2$ (so that the x and y axes lie along the sides and z is still perpendicular).

3. **Start Up of Rolling** *a problem seen many places*

   Consider a thin hoop of mass M and radius R that contacts a surface with coefficient of kinetic friction μ_k at time $t = 0$. It is oriented vertically, that is, with its axis of symmetry oriented horizontally, as in the figures below. The moment of inertia of the hoop is $I = MR^2$.

   (a) When the hoop contacts the surface, it has zero initial velocity and initial angular velocity ω_0, as in the figure below. When does the hoop stop slipping (ie, begin rolling without slipping)? How far does it travel before that time?



(b) Suppose instead that the hoop has initial velocity v_0 in the $+x$ direction without rotating (initial angular velocity vanishes), as in the figure below. What is the speed of the hoop when it starts rolling without slipping?