# PHYS-3202 Homework 1 Due 22 Sept 2021

This homework is due to `https://uwcloud.uwinnipeg.ca/s/wxqoYpEEa8WT2LX` by 10:59PM on the due date. Your file(s) must be in PDF format; they may be black-and-white scans or photographs of hardcopies (all converted to PDF), PDF prepared by LaTeX, or PDF prepared with a word processor *using an equation editor*.

1. **Turbulent Air Resistance**

   Consider an object falling in a uniform gravitational acceleration $g$ against a drag force of magnitude $\lambda v^2$. In this problem, you will want to recall the hyperbolic trig functions and the relationships $\cosh^2\theta - \sinh^2\theta = 1$, $d\cosh\theta/d\theta = \sinh\theta$, and $d\sinh\theta/d\theta = \cosh\theta$.

   (a) Show that the speed of the object as a function of time is

   $$v(t) = \sqrt{\frac{mg}{\lambda}}\tanh\left(\sqrt{\frac{\lambda g}{m}}\,t\right) \ , \tag{1}$$

   where $m$ is the object's mass. Assume that $v = 0$ at $t = 0$. Does this formula agree with the terminal velocity from the lecture notes? *Hint:* You can directly integrate Newton's 2nd law using a hyperbolic trig substitution.

   (b) Now find the distance traveled as a function of time. Check that your answer has the correct units.

2. **Rocket Science**

   Consider a rocket of initial velocity $v_0$ and initial total mass (including fuel) $m_0$ moving linearly in outer space. Recall from class that its velocity at a later time $t$ is $v = v_0 + u\ln(m_0/m)$, where $u$ is the exhaust speed relative to the rocket and $m$ is the mass at time $t$.

   (a) *from Thornton & Marion* What is the ratio $m/m_0$ when the momentum of the rocket is maximized? *Hint:* Remember that the mass of the rocket is changing as it burns and exhausts fuel.

   (b) *from Cline 2.10* Assume the rocket exhausts fuel at a constant rate $\dot{m} = -k$ (until the fuel runs out). Find the displacement as a function of time.

   (c) Finally, assuming that the relative exhaust speed $u$ is constant, what is the mass of the rocket as a function of time if its acceleration $a$ is constant?

3. **Impulse and Bouncing**

   The *impulse* $\vec{I}$ acting on an object is defined as the time integral of the force acting on the object,

   $$\vec{I} \equiv \int dt\,\vec{F} \ . \tag{2}$$

   Impulse is a useful concept when there is a large force acting over a short period of time, such as the normal force between colliding objects. Note that a constant force (like gravity) gives a very small impulse over a short time period.

   (a) Show that the impulse acting on an object over time $\Delta t$ equals the change in momentum of the object over that time.

(b) Box $A$ of mass $M$ sits at rest on a (horizontal) table. Box $B$ of mass $m$ falls directly downward and collides with box $A$ at speed $v$ with coefficient of restitution $e$. What is the (vertical component of) impulse acting on box $B$? (Box $A$ stays motionless on the table.) What type of force causes this impulse?

(c) Now suppose that box $A$ is moving at speed $u$ to the right along the table when box $B$ strikes it. Note that the surfaces of the two boxes are moving horizontally with respect to each other during the collision with coefficient of kinetic friction $\mu_k$. Find the horizontal component of the impulse on box $B$ and use it to find the horizontal component of the velocity of box $B$ after the collision. What is the final horizontal velocity of box $A$?