Intermediate Mechanics PHYS-3202 Final Exam
The University of Winnipeg

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19-21 Dec 2021

Instructions

- This test will be available at noon CST Sun 19 Dec 2021 and is due at noon CST Tues 21 Dec 2021. I will post the exam on the webpage and email it to your preferred email addresses.
- Upload your solutions to `https://uwcloud.uwinnipeg.ca/s/wxqoYpEEa8WT2LX` . **This is the same link as for homework.**
- Submissions should be PDF files that are either scanned hardcopies or prepared with LaTeX or else MS Word with an equation editor for mathematics (*please export your Word file to PDF to submit*). Label your filenames with your first initial, last name, and "exam" (for example `AFrey_exam.pdf`); if you need to break your solution into multiple parts, label them in order with page numbers (`AFrey_exam1.pdf`, `AFrey_exam2.pdf`, etc). See the homework submission instructions on the course outline.
- You may consult any resources linked on the course web page including the textbooks by Idema and by Cline, other assigned readings, lecture notes, and homework solutions. **No other resources are allowed (including calculators, other mathematical software including Maple or Mathematica, etc)**. Note that using other resources or consulting other people, including other students, will be considered cheating and may lead to discipline under the University's Academic Misconduct policy and procedures.
- If you have questions, you may email me, and I will answer as soon as possible. Alternately, I will be available on zoom from 2-3PM 20 Dec and 11AM-noon 21 Dec at the usual zoom meeting for the class lectures for you to ask questions and confirm that I have received your exam solutions.
- This test has 2 pages of questions (3 total pages including cover sheets).
- **Answer all questions briefly and completely.** You may re-use results in multiple problems if helpful, but please reference the first problem where you use them.

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Answer all questions briefly but completely.

Recent Topics:

1. [5 points] A particle approaches a spherical object of radius R. There is a repulsive inverse square force $\vec{F} = k\hat{r}/r^2$ between the incoming particle and the object ($k > 0$). Is the total cross section for the particle to collide physically with the spherical object larger or smaller than πR^2? Explain your answer briefly; you do not need to do any calculations.

2. [15 points] A cube of mass M and uniform density fills the region $0 \le x \le L$, $0 \le y \le L$, $-L/2 \le z \le L/2$. Find its inertia tensor in this coordinate system.

3. [20 points] I am standing on the surface of the earth at colatitude θ. I throw a ball due north with initial speed v at an angle α from the vertical. Show that the ball will land a distance

$$\frac{4\omega v^3}{g^2}\cos^2\alpha\left(\cos\theta\sin\alpha - \frac{1}{3}\sin\theta\cos\alpha\right) \tag{1}$$

to the east of where it would have landed without the Coriolis force, where ω is the angular frequency of the earth's rotation (a negative distance is deflection to the west). *Note:* We take the positive x direction to the east, positive y to the north, and positive z upward, so the trajectory of the ball ignoring the Coriolis force is

$$x = 0\ ,\quad y = v\sin\alpha t\ ,\quad z = v\cos\alpha t - \frac{1}{2}gt^2\ . \tag{2}$$

The angular velocity vector of the earth is $\vec{\omega} = \omega(\sin\theta\hat{y} + \cos\theta\hat{z})$.

Cumulative:

4. An *exoplanet* is a planet orbiting another star. Consider an exoplanet with mass $M = 2M_\oplus$, radius $R = 4R_\oplus$, and angular velocity $\omega = \omega_\oplus/2$, where $M_\oplus, R_\oplus, \omega_\oplus$ are the values of those quantities for the earth. Fill in the blank for each sentence below with the correct choice and explain your answer briefly. You may approximate both the earth and the exoplanet as perfect spheres.

   (a) [5 points] A satellite orbits the exoplanet with the same orbital semi-major axis as the moon has around the earth. The period of the exoplanetary satellite orbit is _____ the period of the earth's moon's orbit.
   A. less than B. equal to C. greater than

   (b) [5 points] The difference between the effective gravitational acceleration at the poles and at the equator of the exoplanet is _____ the same difference for the earth.
   A. less than B. equal to C. greater than

   (c) [5 points] The magnitude of the Coriolis force for horizontal motion with a velocity of 1 m/s at the pole of the exoplanet is _____ the magnitude of the Coriolis force for the same motion at the north pole of the earth.
   A. less than B. equal to C. greater than

5. An underdamped harmonic oscillator of mass m has motion in 1 dimension given by

$$x(t) = Ae^{-\gamma t + i\omega t} + A^* e^{-\gamma t - i\omega t} \ , \tag{3}$$

where A, γ, and ω are all constants and the star represents the complex conjugate. The oscillator has initial conditions $x(0) = x_0$, $\dot{x}(0) = 0$. No external force acts on the oscillator.

(a) [5 points] Find the complex constant A.

(b) [5 points] The motion of the oscillator can be written in the form

$$x(t) = ae^{-bt}\cos(\alpha t + \theta) \ , \tag{4}$$

where a, b, α, and θ are constants. Find a, b, α, and θ in terms of x_0, γ, and ω.

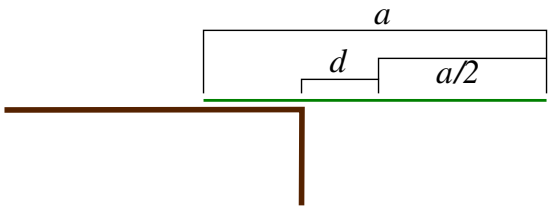
(c) [5 points] What is the total energy of the oscillator at time $t = 0$?

6. [10 points] Consider a closed elliptical orbit in an inverse square force with potential energy $V(r) = k/r$, $k < 0$. Show that the time derivative of the radial position can be written as

$$\dot{r} = -\frac{1}{r}\sqrt{\frac{|k|}{m}}\sqrt{(e^2-1)\frac{r^2}{\ell} + 2r - \ell} \ , \tag{5}$$

where m is the mass of the object in orbit, e is the orbit eccentricity, and ℓ is the orbit semi-latus rectum. This would allow us to find the time along the orbit. *Hint:* See the lecture notes where we derived the formula for the orbit, including the relationships between energy, angular momentum, and the orbit parameters.

7. At time $t = 0$, a rigid uniform rectangular lamina (flat sheet) with sides of length a and b and mass M lies horizontally off the edge of a table as shown in the figure. The lamina's center of mass is a distance d beyond the edge of the table, and the side of length b is parallel to the edge of the table.



(a) [5 points] Find the moment of inertia of the lamina around the edge of the table.

(b) [5 points] Find the torque due to gravity around the edge of the table.

(c) [10 points] Starting at $t = 0$, the lamina begins to rotate around the edge of the table. Assuming the normal force on the lamina acts at the edge of the table, find the magnitude of the normal force at $t = 0$. *Hint:* relate the angular acceleration of the lamina to the acceleration of its center of mass.