Intermediate Mechanics PHYS-3202 Mid-Term Test

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27-29 Oct 2021

Instructions

- This test will be available at noon CDT Weds 27 Oct 2021 and is due at 11:20AM CDT Fri 29 Oct 2021. I will post the exam on the webpage and email it to your preferred email addresses.
- Upload your solutions to `https://uwcloud.uwinnipeg.ca/s/wxqoYpEEa8WT2LX` . **This is the same link as for homework.**
- Submissions should be PDF files that are either scanned hardcopies or prepared with LaTeX or else MS Word with an equation editor for mathematics (*please export your Word file to PDF to submit*). Label your filenames with your first initial, last name, and "test" (for example `AFrey_test.pdf`); if you need to break your solution into multiple parts, label them in order with page numbers (`AFrey_test1.pdf`, `AFrey_test2.pdf`, etc). See the homework submission instructions on the course outline.
- You may consult any resources linked on the course web page including the textbooks by Idema and by Cline, other assigned readings, lecture notes, and homework solutions. **No other resources are allowed (including calculators, other mathematical software including Maple or Mathematica, etc)**. Note that using other resources or consulting other people, including other students, will be considered cheating and may lead to discipline under the University's Academic Misconduct policy and procedures.
- If you have questions, you may email me, and I will answer as soon as possible. Alternately, I will be available on zoom from 1-2PM Oct 28 and 10:30-11:20AM Oct 29 at the usual zoom meeting for the class lectures for you to ask questions and confirm that I have received your exam solutions.
- This test has 2 pages of questions (3 total pages including cover sheets).
- **Answer all questions briefly and completely.** You may re-use results in multiple problems if helpful, but please reference the first problem where you use them.

Answer all questions briefly but completely.

1. For each part, choose all correct answer(s) from the options given and explain your answer in no more than two lines. There may be one or more correct answers for each part. (The variable k is a constant of appropriate units in all parts below.)
   (a) [5 points] A central force has the form $\vec{F} = -f(r)\hat{r}/r^2$. For which functions $f(r)$ below is $\vec{F}$ conservative?
   A. $f(r) = 1$ B. $f(r) = e^{-kr}$ C. $f(r) = (1 + kr)e^{-kr}$ D. $f(r) = 4r^5$
   (b) [5 points] For motion in the x direction only, which of the following forces is conservative? Here v denotes velocity, not speed.
   A. $F = -k$ B. $F = -kx$ C. $F = -kv$ D. $F = -kxv$
   (c) [5 points] For motion in three dimensions, which of the following forces is conservative?
   A. $\vec{F} = ky\hat{\imath} - kx\hat{\jmath}$ B. $\vec{F} = kz^3\hat{k}$ C. $\vec{F} = ke^{kx}\hat{\jmath}$ D. $\vec{F} = ky\hat{\imath} + kx\hat{\jmath}$
   (d) [5 points] $\vec{F}$ is a conservative force with potential energy $V(\vec{r})$. Which of the following give the work done by the force on an object moving from $\vec{r}_1$ to $\vec{r}_2$?
   A. $V(\vec{r}_1) - V(\vec{r}_2)$ B. $V(\vec{r}_2) - V(\vec{r}_1)$ C. $\int dt\vec{F}$ D. $\int_{\vec{r}_1}^{\vec{r}_2} d\vec{r} \cdot \vec{F}$

2. In the class notes, we gave the solution for the motion of an object in gravity with a linear air resistance force $\vec{F} = -m\gamma\vec{v}$ as
$$z(t) = z_0 + \left(\frac{v_0}{\gamma} + \frac{g}{\gamma^2}\right)\left(1 - e^{-\gamma t}\right) - \frac{gt}{\gamma}\ , \quad (1)$$
   where z is the vertical coordinate increasing upward and g is the acceleration due to gravity (which points downward). Assume that $v_0 \gg g/\gamma > 0$.
   (a) [10 points] Show that the object reaches the top of the trajectory at $\gamma t \approx \ln(\gamma v_0/g) + g/\gamma v_0$. *Hint:* use the Taylor series $\ln(1 + x) = x + \cdots$ for $x \ll 1$.
   (b) [10 points] Using just the larger term in the approximation $\gamma t \approx \ln(\gamma v_0/g)$, show that the maximum height the object reaches is $z \approx z_0 + v_0/\gamma$. Compare this to the maximum height reached in the absence of air resistance. *Hint:* Recall $x \gg \ln(x)$ for $x \gg 1$.

3. Consider a critically damped harmonic oscillator with mass m, natural frequency ω_0, and damping coefficient $\gamma = \omega_0$. The oscillator has initial conditions $x = L, \dot{x} = 0$ at $t = 0$.
   (a) [5 points] Find the solution $x(t)$ for motion of the oscillator. Make sure to use the initial conditions to determine constants of integration.
   (b) [10 points] Find the total kinetic plus potential energy of the oscillator at time t. Is this more or less than the total energy at $t = 0$? Why?
   (c) [5 points] Without using the solution for $x(t)$, find the work done on the oscillator by the damping force from $t = 0$ to $t = \infty$. You may use the fact that $x \to 0$ as $t \to \infty$.

4. The planet Saturn (mass M) has many moons, including Dione and Iapetus.
   (a) [10 points] Dione has an approximately circular orbit of radius R. Find the period of Dione's orbit in terms of M, R, and physical constants.
   (b) [10 points] Dione's orbital radius is $R = 4 \times 10^5$ km and has period 3 days. Iapetus has a pericenter distance of 3.5×10^6 km and apocenter distance of 3.7×10^6 km. What is the period of Iapetus's orbit? Give your answer to one significant digit in days.

5. [20 points] Consider a projectile of mass m launched from the origin with initial velocity $\vec{v} = v_x\hat{\imath} + v_y\hat{\jmath}$, where y increases upward (y is the vertical coordinate). Calculate the angular momentum of the projectile as a function of time and show that its time derivative is equal to the torque due to gravity. You may assume that the projectile is near the surface of the earth (so the acceleration due to gravity is constant) and air resistance is negligible.