

## PHYS-4602 Homework 9 Due 31 Mar 2022

This homework is due to <https://uwcloud.uwinnipeg.ca/s/yPzo5AdxJx4oCMn> by 10:59PM on the due date. Your file(s) must be in PDF format; they may be black-and-white scans or photographs of hardcopies (all converted to PDF), PDF prepared by LaTeX, or PDF prepared with a word processor *using an equation editor*.

### 1. Uniform Gravitational Field *parts of Griffiths 9.6 and 9.7*

Consider a ball of mass  $m$  that feels a uniform gravitational acceleration  $g$  in the  $-x$  direction, as by the surface of the earth. Assume that the surface of the earth is at  $x = 0$  and forms an infinite potential barrier.

- First, write down what the potential energy is as a function of  $x$ .
- Use the WKB approximation to find the allowed energies of the bouncing ball. Find the approximate ground state and first excited state energies in Joules to two significant digits for a neutron (mass  $m = 1.7 \times 10^{-27}$  kg). This can actually be measured for ultracold neutrons.
- The *exact* solution of the Schrödinger equation is given by the Airy function

$$\psi(x) = CAi \left[ \left( \frac{2m^2g}{\hbar^2} \right)^{1/3} \left( x - \frac{E}{mg} \right) \right], \quad (1)$$

where  $C$  is a normalization constant and  $E$  is quantized so  $\psi(0) = 0$ . Denote the zeros of  $Ai(z)$  by  $a_k$  ( $k = 1, 2, \dots$  with  $|a_1| < |a_2| < \dots$ ) and find the energy eigenvalues in terms of the  $a_k$ . What are the ground and first excited state energies for a neutron? You will need to look up values of  $a_k$  at the Digital Library of Mathematical Functions (DLMF) at <http://dlmf.nist.gov/9.9>.

- Show that the energy eigenvalues match the WKB result in the limit of large quantum number. *Hint:* You can use the asymptotic form of the Airy function itself (either in Griffiths or in the DLMF) or that of the zeros (from the DLMF).

### 2. Ionizing an Atom *from Griffiths 9.18*

Imagine a hydrogen atom in a small electric field; the electron feels a linear potential from the field, which eventually becomes less than the ground state energy, so it can tunnel out of the atom. In this problem, consider a simple 1D model of this system, with potential

$$V(x) = \begin{cases} \infty, & x < -a \\ -V_0, & -a < x < 0 \\ -\alpha x, & x > 0 \end{cases}. \quad (2)$$

- Suppose the square well is very deep, so  $V_0 \gg \hbar^2/ma^2$ . In the absence of the electric field ( $\alpha = 0$ ), what is the approximate ground state energy  $E$ ? If the electron were a classical particle with this kinetic energy, what would be its speed? *Hint:* You can think of this as the energy of the first odd eigenfunction of a finite square well of width  $2a$  or you can approximate the potential as nearly an infinite square well.
- Show that the lifetime of the atom in the presence of the field is  $\ln \tau = A|E|^{3/2} + B$ , where  $A$  and  $B$  are constants. Then find  $A$  and  $B$  (you may need your results from part (a)). *Hint:* the lifetime is the reciprocal of the transition rate defined as in the notes.