PHYS-4602 Homework 8 Due 24 Mar 2022

This homework is due to `https://uwcloud.uwinnipeg.ca/s/yPzo5AdxJx4oCMn` by 10:59PM on the due date. Your file(s) must be in PDF format; they may be black-and-white scans or photographs of hardcopies (all converted to PDF), PDF prepared by LaTeX, or PDF prepared with a word processor *using an equation editor.*

1. **Sharp Kick**

   Consider a particle initially in the ground state of a 1D infinite square well with potential

   $$V(x) = \left\{ \begin{array}{ll} 0 & 0 < x < a \\ \infty & \text{otherwise} \end{array} \right. . \tag{1}$$

   At time $t = 0$, the particle receives a kick in the form of a time-dependent potential $\alpha \cos(\pi x/a)\delta(t)$ for small α. What is the probability that the particle is in the first excited state after $t = 0$? (*Hint:* you need to generalize the limits of the time integral slightly for the transition amplitude.)

2. **A Pair of Electrons** *based on a problem from McGill physics*

   Two electrons are localized at well-separated lattice sites, so they can be treated as distinguishable particles. The two electrons interact with each other, and only the first electron experiences a magnetic field. The Hamiltonian for $t \leq 0$ is $H_0 = A\vec{S}_1 \cdot \vec{S}_2$, where $\vec{S}_j$ is the spin of electron j and A is a positive constant. For $t > 0$, the Hamiltonian is $H = H_0 + B(t)S_{1,z}$, where $B(t) = B_0 \sin(\omega t)$ and $B_0 \ll A\hbar$. (The B term represents the magnetic field on the first electron.)

   (a) List the ground and excited states for Hamiltonian H_0. The system is in the H_0 ground state at $t = 0$, and we measure it at a later time t. What possible transitions between H_0 eigenstates could have occurred at first order in perturbation theory? *Hint:* See our discussion of the hydrogen fine and hyperfine structure for how to diagonalize H_0.

   (b) For any possible transition you found in the previous part, what is the transition probability at time t? If $t \to \infty$, what value of the perturbation frequency ω gives a nonzero transition rate?

3. **Variational Calculations**

   (a) Consider a particle moving in 1D in a potential $V(x) = \alpha|x|$. Find the best possible upper bound on the ground state energy using a gaussian trial wavefunction.

   (b) Consider a particle moving in the 1D interval $0 \leq x < L$ with periodic boundary conditions on the wavefunction $\psi(0) = \psi(L)$. The particle experiences a potential $V(x)$. Show that the ground state energy of this system is less than or equal to the average value of $V(x)$ over the range $0 \leq x < L$.

   (c) *from Griffiths 8.6* Consider a Hamiltonian $H = H_0 + H_1$, where H_0 is exactly solvable and H_1 is small in some sense. Prove that first-order perturbation theory always overestimates the true ground state energy. That is, show that the ground state energy calculated in first-order perturbation theory is greater than (or equal to) the true ground state energy.