PHYS-4602 Homework 6 Due 1 Mar 2022

This homework is due to `https://uwcloud.uwinnipeg.ca/s/yPzo5AdxJx4oCMn` by 10:59PM on the due date. Your file(s) must be in PDF format; they may be black-and-white scans or photographs of hardcopies (all converted to PDF), PDF prepared by LaTeX, or PDF prepared with a word processor *using an equation editor.*

Note that I cannot give an extension on this assignment due to the midterm test.

1. **Center of the Box**

   We insert a particle into an infinite square well with boundaries at $x = \pm a$ in such a way that it is highly localized at the origin, so we can approximate its state as a position eigenket $|x = 0\rangle$. Write $|x = 0\rangle$ as a superposition of energy eigenstates.

2. **Momentum Differentiates the Position Operator**

   Use the rule that $[A, B^n] = n[A, B]B^{n-1}$ when $[A, B]$ commutes with B to prove that $[p, f(x)] = -i\hbar df/dx$, where x and p are 1D position and momentum operators with $[p, x] = -i\hbar$. Assume $f(x)$ can be written as a Taylor series. (Please refer to homework assignment #1 problem 4.)

3. **Gaussian Wavepacket**

   Here we consider the Gaussian wavepacket in 1D at a single instant $t = 0$, ignoring its time evolution. The state is

   $$|\psi\rangle = \int_{-\infty}^{\infty} dx\ Ae^{-ax^2}|x\rangle\ . \tag{1}$$

   Some of these results may be useful on future assignments.

   (a) Find the normalization constant A. *Hint:* To integrate a Gaussian, consider its square. When you square it, change the dummy integration variable to y, then change the integral over $dxdy$ to plane polar coordinates. The textbook cover also has a formula for Gaussian integrals.

   (b) Since the wavefunction is even, $\langle x\rangle = 0$. Find $\langle x^2\rangle$. *Hint:* You can get a factor of x^2 next to the Gaussian by differentiating it with respect to the parameter a.

   (c) Write $|\psi\rangle$ in the momentum basis. *Hint:* If you have a quantity $ax^2 + bx$ somewhere, you may find it useful to write it as $a(x + b/2a)^2 - b^2/4a$ by completing the square. Then shift integration variables so it looks like you have a Gaussian again.

   (d) Find $\langle p\rangle$ and $\langle p^2\rangle$ and show that this state saturates the Heisenberg uncertainty principle. You should not have to do any integrations.

4. **Harmonic Oscillator Matrix Elements**

   Calculate the matrix elements $\langle n|x|n'\rangle$ and $\langle n|p^2|n'\rangle$ for $|n\rangle, |n'\rangle$ stationary states of the harmonic oscillator. You *must* use Dirac and operator notation and *may not* carry out any integrals.

5. **Previous Midterm Multiple Choice**

   Choose **all** correct answers for each part. Explain your answers **very briefly**.

(a) Without solving the characteristic equation, which of the following can **not** be eigenvalues of the matrix

$$\begin{bmatrix} 1 & 0 & 2i \\ 0 & 1 & 0 \\ -2i & 0 & 1 \end{bmatrix} \quad ? \tag{2}$$

A. 3 B. $1 + 2i$ C. -1 D. 0

(b) The stationary states of an electron in the Coulomb potential can be described either by quantum numbers $|n, \ell, m, s, m_s\rangle$ or quantum numbers $|n, j, m_j, \ell, s\rangle$, and the energy depends only on n. Which of the following are true?

A. The Hamiltonian commutes with the spin operator $\vec{S}^2$

B. The linear superposition $(|n, \ell = 1, m = 0, s, m_s\rangle + |n, \ell = 0, m = 0, s, m_s\rangle)/\sqrt{2}$ is an energy eigenstate for any allowed n, s, m_s.

C. The total z angular momentum operator J_z commutes with the total orbital angular momentum operator $\vec{L}^2$

(c) Which of the following equals the Hadamard operator $\mathbb{H}$?

A. $|1\rangle\langle 0| + |0\rangle\langle 1|$ B. $|0\rangle\langle 0| - |1\rangle\langle 1|$ C. $|+\rangle\langle +| + |-\rangle\langle -|$ D. $|+\rangle\langle 0| + |-\rangle\langle 1|$

(d) If I state that whether Schrödinger's cat lives or dies is predetermined by secret physics of the radioactive nucleus before I close it into the box, what type of theory of quantum mechanics am I expressing?

A. Hidden Variables Theory B. Copenhagen Interpretation C. Many Worlds Theory D. Bell's Theory