PHYS-4602 Homework 5 Due 17 Feb 2022

This homework is due to `https://uwcloud.uwinnipeg.ca/s/yPzo5AdxJx4oCMn` by 10:59PM on the due date. Your file(s) must be in PDF format; they may be black-and-white scans or photographs of hardcopies (all converted to PDF), PDF prepared by LaTeX, or PDF prepared with a word processor *using an equation editor*.

1. **Coherent Teleportation**

   Consider a system of 3 qubits. We will demonstrate the existence of *coherent teleportation*, which transfers a state $|\psi\rangle$ from qubit 1 to qubit 3 without requiring measurement.

   For notation, a subscript i on a 1-qubit operator means it acts on qubit i, so $\mathbb{H}_i$ is the Hadamard operator acting on qubit i. We also define $\Gamma_{i,j}$ as the CNOT operator acting on qubit j with qubit i as control. For example, $\Gamma_{1,2}$ is the usual CNOT operator that reverses qubit 2 if qubit 1 is $|1\rangle$, while $\Gamma_{3,1}$ reverses qubit 1 if qubit 3 is $|1\rangle$, etc. Finally, we denote $|\pm\rangle = (|0\rangle \pm |1\rangle)/\sqrt{2}$ for any qubit.

   (a) Show that
   $$\frac{1}{\sqrt{2}}\left(|+\rangle_1|+\rangle_2 + |-\rangle_1|-\rangle_2\right) = \frac{1}{\sqrt{2}}\left(|0\rangle_1|0\rangle_2 + |1\rangle_1|1\rangle_2\right) \ . \tag{1}$$

   (b) We wish to define a new operator $\Delta_{i,j}$ that takes $\Delta_{i,j}|+\rangle_i|0\rangle_j = |+\rangle_i|+\rangle_j$ and $\Delta_{i,j}|-\rangle_i|0\rangle_j = |-\rangle_i|-\rangle_j$. Show that $\Delta_{i,j} \equiv \mathbb{H}_i\mathbb{H}_j\Gamma_{i,j}\mathbb{H}_i$ obeys these equations. With this definition, find $\Delta_{i,j}|+\rangle_i|1\rangle_j$ and $\Delta_{i,j}|-\rangle_i|1\rangle_j$.

   (c) Show that $\Delta_{i,j}$ is not a cloning operator. *Hint:* If $|\psi\rangle = a|+\rangle + b|-\rangle$, show that $\Delta_{1,2}|\psi\rangle_1|0\rangle_2 \neq |\psi\rangle_1|\psi\rangle_2$ by comparing both sides.

   (d) Let $|\psi\rangle = a|0\rangle + b|1\rangle$ be a general qubit. Show that
   $$\Gamma_{3,2}\Delta_{1,2}\Gamma_{1,3}|\psi\rangle_1|0\rangle_2|0\rangle_3 = \frac{1}{\sqrt{2}}\Big(|0\rangle_1|0\rangle_2 + |1\rangle_1|1\rangle_2\Big)|\psi\rangle_3 \ . \tag{2}$$

2. **Quantum Reality or Not**

   To answer this question, you will need to watch the video of Sidney Coleman's famous lecture "Quantum Mechanics In Your Face" at `http://media.physics.harvard.edu/video/?id=SidneyColeman_QMIYF` or `https://www.youtube.com/watch?v=EtyNMlXN-sw` . (This is about an hour and supplements the reading, which is not long this week; the transcript is at `https://arxiv.org/pdf/2011.12671.pdf`.)

   (a) The Bell experiment considers 2 distinguishable spin 1/2 particles in the singlet ($s = 0$) total spin state. If $\hat{a}$ and $\hat{b}$ are two unit vectors, show that
   $$\left\langle \left(\hat{a}\cdot\vec{S}^{(1)}\right)\left(\hat{b}\cdot\vec{S}^{(2)}\right)\right\rangle = -\frac{\hbar^2}{4}\hat{a}\cdot\hat{b} \ . \tag{3}$$

   Hint: Think about a convenient choice of axes and remember that the spin operators are given in matrix form as $S_i \simeq (\hbar/2)\sigma_i$ in terms of the Pauli matrices.

   (b) Three electrons are prepared in the so-called "GHZM" spin state
   $$|\psi\rangle = \frac{1}{\sqrt{2}}(|\uparrow\rangle_1|\uparrow\rangle_2|\uparrow\rangle_3 - |\downarrow\rangle_1|\downarrow\rangle_2|\downarrow\rangle_3) \tag{4}$$

described in the video. Show that $|\psi\rangle$ is an eigenstate of the operator $S_x^{(1)} S_y^{(2)} S_y^{(3)}$ and find the eigenvalue.

3. **Copenhagen vs Many-Worlds** *sample from previous midterm*

   Quantum teleportation transfers an unknown state $|\psi\rangle$ from one qubit to another at a distance. This process involves two measurements.

   (a) In the Copenhagen interpretation of quantum mechanics, is quantum teleportation described by a unitary operation? Explain very briefly.

   (b) In the many worlds interpretation of quantum mechanics, is quantum teleportation described by a unitary operation? Explain very briefly.