PHYS-4602 Homework 2 Due 27 Jan 2022

This homework is due to `https://uwcloud.uwinnipeg.ca/s/yPzo5AdxJx4oCMn` by 10:59PM on the due date. Your file(s) must be in PDF format; they may be black-and-white scans or photographs of hardcopies (all converted to PDF), PDF prepared by LaTeX, or PDF prepared with a word processor *using an equation editor.*

1. **Measurement vs Time Evolution** *a considerable revision of Griffiths 3.33*

   Suppose a system has observable A with eigenstates $|a_1\rangle, |a_2\rangle$ of eigenvalues a_1, a_2 respectively and Hamiltonian H with eigenstates $|E_1\rangle, |E_2\rangle$ of energies E_1, E_2 respectively. The eigenstates are related by

   $$|a_1\rangle = \frac{1}{5}\left(3|E_1\rangle + 4|E_2\rangle\right)\ ,\quad |a_2\rangle = \frac{1}{5}\left(4|E_1\rangle - 3|E_2\rangle\right)\ . \tag{1}$$

   Suppose the system is measured to have value a_1 for A initially. Each of the following parts asks about a different possible set of subsequent measurements.

   (a) What is the probability of measuring energy E_1 immediately after the first measurement? Assuming we do get E_1, what is the probability of measuring a_1 again if we measure A again immediately after the measurement of energy?

   (b) Instead, consider immediately measuring A again after the first measurement. What are the probabilities for observing a_1 and a_2?

   (c) Finally, consider making the first measurement and then allowing the system to evolve for time t. If we then measure energy, what is the probability of finding energy E_1? If we instead measured A again, what is the probability we find a_1 again?

2. **Oscillating Spin**

   Consider a spin-1/2 particle like an electron. In terms of the S_z eigenstates $|\uparrow\rangle$ and $|\downarrow\rangle$, the eigenstates of the S_y operator are

   $$|\rightarrow\rangle = \frac{1}{\sqrt{2}}\left(|\uparrow\rangle + i|\downarrow\rangle\right)\ ,\quad |\leftarrow\rangle = \frac{1}{\sqrt{2}}\left(|\uparrow\rangle - i|\downarrow\rangle\right)\ , \tag{2}$$

   where $|\rightarrow\rangle$ has eigenvalue $\hbar/2$ and $|\leftarrow\rangle$ has eigenvalue $-\hbar/2$. The electron is placed in a magnetic field along y, so the Hamiltonian is $H = -\gamma B S_y$. The electron has initial state $|\uparrow\rangle$.

   (a) What is the probability of finding $-\hbar/2$ as the result of a measurement of S_z at time t?

   (b) Find the expectation value $\langle S_y\rangle$ as a function of time.

   (c) As a matrix written with respect to the S_z eigenbasis $\{|\uparrow\rangle, |\downarrow\rangle\}$,

   $$S_y \simeq \frac{\hbar}{2}\begin{bmatrix} 0 & -i \\ i & 0 \end{bmatrix}\ . \tag{3}$$

   Show that the time evolution operator for this Hamiltonian can be written

   $$U(t) \simeq \cos\left(\frac{\gamma B t}{2}\right)\begin{bmatrix} 1 & 0 \\ 0 & 1 \end{bmatrix} + i\sin\left(\frac{\gamma B t}{2}\right)\begin{bmatrix} 0 & -i \\ i & 0 \end{bmatrix} \tag{4}$$

   in the S_z eigenbasis.

3. **3-Particle States** *from some Griffiths problems*

   Consider three particles, each of which is in one of the single-particle states $|\alpha\rangle$, $|\beta\rangle$, or $|\gamma\rangle$, which are orthonormal.

   (a) If the particles are bosons, write down the state where one particle is in each of $|\alpha\rangle$, $|\beta\rangle$, and $|\gamma\rangle$. *Hint*: This state must be symmetric under the exchange of *any* pair of the bosons.

   (b) Write down all possible 3-particle states (including normalization) with two particles in the same 1-particle state and the third particle in a different 1-particle state, still in the case that the particles are indistinguishable bosons.

   (c) How many linearly independent states can you form if the particles are fermions? Write down all the possible linearly independent states. *Hint*: Similarly to the above, these states must be antisymmetric under the exhange of *any* pair of the fermions.