PHYS-4602 Homework 9 Due 7 Apr 2022

This homework is due to `https://uwcloud.uwinnipeg.ca/s/yPzo5AdxJx4oCMn` by 10:59PM on the due date. Your file(s) must be in PDF format; they may be black-and-white scans or photographs of hardcopies (all converted to PDF), PDF prepared by LaTeX, or PDF prepared with a word processor *using an equation editor.*

1. **Scattering from a Spherical Shell Potential** *from Griffiths & Schroeter 10.13*

   Consider scattering from a spherical shell potential $V(r) = \alpha\delta(r-a)$ for constant a, α. Work in the Born approximation. Note that the scattering amplitude $f(\theta, \phi) = f(\theta)$ depends only on the scattering angle due to spherical symmetry. The scattered particle has mass m.

   (a) Find the scattering amplitude for low energy scattering $ka \ll 1$.

   (b) Use the spherical symmetry to find the scattering amplitude as a function of the incoming wave energy E for all energies. Show that you find your previous result in the low energy limit.

   (c) What are the differential cross section and total cross section in the low energy limit?

   (d) Find the differential cross section for a given energy E, which might not be small. For how many different scattering angles will the differential cross section vanish? *Hint:* write your answer in terms of the "floor" function, where floor(x) is the largest integer $\leq x$.

2. **Yukawa Potential Scattering** *extended from Griffiths & Schroeter 10.11 & 10.12*

   The attractive Yukawa potential (in 3D) is a central potential given by

$$V(r) = -\beta \frac{e^{-\mu r}}{r} \ , \tag{1}$$

   where β, μ are constants. This is the potential for electromagnetism with massive photons and a (very oversimplified) model for the strong force between neutrons and protons.

   (a) Find the scattering amplitude of a particle of mass m as a function of the incoming wavevector k and scattering angle θ at first order in the Born approximation (see example 10.5 but show how to evaluate the integral).

   (b) Find the differential cross section and total cross section in the Born approximation using your result from above. Write your answers in terms of the incoming energy E and (when appropriate) the scattering angle θ.

   (c) Using your result from the previous part, find the total cross section at low energies (in the Born approximation). Verify that it is $4\pi|f|^2$ as expected in this limit.