PHYS-3203 Homework 8 Due 30 Mar 2022

This homework is due to `https://uwcloud.uwinnipeg.ca/s/QGK3eGfDRgND6sC` by 10:59PM on the due date. Your file(s) must be in PDF format; they may be black-and-white scans or photographs of hardcopies (all converted to PDF), PDF prepared by LaTeX, or PDF prepared with a word processor *using an equation editor.*

1. **Some Scalar Products** *inspired by a problem in Hartle*

   In some frame, the components of two 4-vectors are

   $$a^\mu = (2,0,0,1)c \text{ and } b^\mu = (5,4,3,0)c \ , \tag{1}$$

   where c is the speed of light as usual.

   (a) Find a^2, b^2, and $a \cdot b$.

   (b) Does there exist another inertial frame in which the components of a^μ are $(c,0,0,c)$? What about b^μ? Explain your reasoning.

   (c) Could either of these 4-vectors represent a 4-velocity (for a normal massive particle)? Explain.

2. **Lightlike Vectors**

   Now consider lightlike 4-vectors f^μ and g^μ.

   (a) If f^μ and g^μ are orthogonal ($f \cdot g = 0$), prove that they are parallel ($f^\mu \propto g^\mu$).

   (b) Is the 4-vector $f^\mu + g^\mu$ spacelike, timelike, or lightlike? Assume that both $f^0 > 0$ and $g^0 > 0$.

3. **SN1987A and Neutrino Masses**

   On 23 Feb 1987, astronomers were startled by the observation of a new supernova in the Large Magellanic Cloud, a satellite galaxy of our Milky Way. However, the first observation of this supernova was several hours earlier by the detection of neutrinos, which was confirmed by two detectors. (The neutrinos arrived before the light because light is trapped for a while by all the matter inside the exploding star.) The fact that the neutrinos all arrived within a few seconds of each other after traveling for more than 100,000 lightyears allows us to put tight constraints on the mass of the neutrino. This problem will guide you through a real calculation of this limit.

   (a) Show that a neutrino with energy $E \gg mc^2$ has a speed approximately given by

   $$\frac{|\vec{u}|}{c} \approx 1 - \frac{1}{2}\left(\frac{mc^2}{E}\right)^2 \ . \tag{2}$$

   Hint: We gave formulas in class for energy both in terms of the spatial momentum and in terms of the speed. Try looking at those. Then you will need to make an expansion in powers of mc^2/E.

   (b) Light (once free of the matter in the supernova) takes a time $t_0 = 5.3 \times 10^{12}$ s to travel from SN1987A to the earth. How long would a neutrino of energy E take to reach earth from the supernova? Work to the lowest non-trivial order in mc^2/E and give the answer in terms of t_0, m, c, and E. Use (2).

(c) The Kamioka detector in Japan detected several neutrinos. The first arrived with energy 21.3 MeV, and another with energy 8.9 MeV arrived 0.303 s later. Assuming that the second neutrino left the supernova no more than 1 s before the first, what is the maximum neutrino mass m? For simplicity, we are ignoring the possible error in the measurements. *Hint:* The observation time of each neutrino is its emission time plus its travel time; take the difference of these and be careful of signs.

For your interest, these neutrino measurements were made by a predecessor experiment to one of the experiments that led to the 2015 Nobel Prize in Physics.