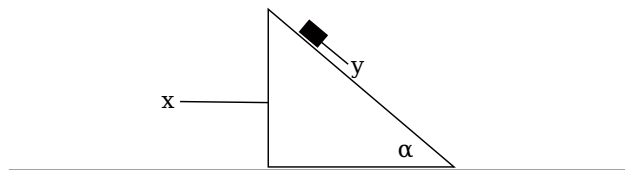


PHYS-3203 Homework 5 Due 2 Mar 2022

This homework is due to <https://uwcloud.uwinnipeg.ca/s/QGK3eGfDRgND6sC> by 10:59PM on the due date. Your file(s) must be in PDF format; they may be black-and-white scans or photographs of hardcopies (all converted to PDF), PDF prepared by LaTeX, or PDF prepared with a word processor *using an equation editor*.

1. Box on a Wedge Hamiltonian Version

Consider again the box of mass m sliding down a wedge of mass M on a frictionless horizontal surface. See the figure



On the previous assignment, you should have found that the Lagrangian is

$$L = \frac{1}{2}M\dot{x}^2 + \frac{1}{2}m(\dot{x}^2 + \dot{y}^2 - 2\dot{x}\dot{y}\cos\alpha) + mgy\sin\alpha. \quad (1)$$

(a) Show that the velocities and canonical momenta are related by

$$p_x = (M + m)\dot{x} - m\dot{y}\cos\alpha, \quad p_y = m(\dot{y} - \dot{x}\cos\alpha) \quad (2)$$

and

$$\dot{x} = \frac{p_x + p_y\cos\alpha}{M + m\sin^2\alpha}, \quad \dot{y} = \frac{p_y}{m} + \frac{p_x + p_y\cos\alpha}{M + m\sin^2\alpha}\cos\alpha. \quad (3)$$

(b) Find the Hamiltonian. *Hint:* note that the term in parenthesis in the Lagrangian can be written $p_y^2/m^2 + \dot{x}^2\sin^2\alpha$.

(c) Name two conserved quantities in this system.

2. Hamiltonian Central Force Motion *expanded from Kibble & Berkshire*

Consider an object of mass m moving in 3D with a central conservative force of potential energy $V(r)$. Note that we learned last semester that the kinetic energy in spherical coordinates is

$$T = \frac{1}{2}m(\dot{r}^2 + r^2\dot{\theta}^2 + r^2\sin^2\theta\dot{\phi}^2). \quad (4)$$

(a) Write the Hamiltonian for this object in spherical polar coordinates.

(b) You should see that the azimuthal angle ϕ is cyclic. Assuming motion is confined to the equatorial plane, find the effective potential for radial motion. Use this to argue that $p_\phi = J_z$, the z component of angular momentum.

(c) Define the square angular momentum

$$\vec{J}^2 = m^2r^4(\dot{\theta}^2 + \sin^2\theta\dot{\phi}^2). \quad (5)$$

Write \vec{J}^2 in terms of canonical momenta and show that it is conserved, even though θ is not cyclic. *Hint:* Look at the class notes for the time dependence of a general function of positions and canonical momenta.

3. Average Height

A particle of mass m is moving near the surface of the earth with gravitational potential energy $V = mgz$, where z is the vertical direction. Assume the particle has energy E .

- (a) Use the virial theorem to find its average height (value of z) over time?
- (b) If the particle starts at rest at its maximum height, write the height z as a function of time. Assuming the particle stops when it reaches the ground at $z = 0$, calculate the time average of z . Is it the same as the answer from the previous part?