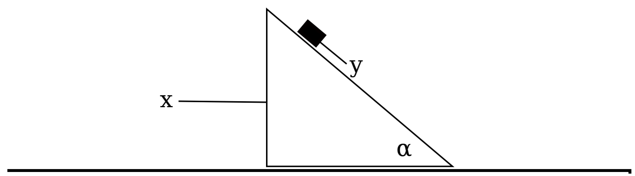
PHYS-3203 Homework 3 Due 2 Feb 2022

This homework is due to `https://uwcloud.uwinnipeg.ca/s/QGK3eGfDRgND6sC` by 10:59PM on the due date. Your file(s) must be in PDF format; they may be black-and-white scans or photographs of hardcopies (all converted to PDF), PDF prepared by LaTeX, or PDF prepared with a word processor *using an equation editor.*

Note that I cannot give an extension on this assignment due to the midterm test.

1. **Box on a Wedge**

   A triangular wedge of mass M is able to slide frictionlessly on a horizontal surface, and a box of mass m can slide frictionlessly down the wedge as in the figure below. The incline of the wedge makes an angle α with the horizontal.

   (a) Write the Lagrangian for this system in terms of x, the displacement of the wedge along the horizontal surface, and y, the displacement of the box down the incline.

   (b) Using the Euler-Lagrange equations, find the acceleration of the wedge as the box slides down the incline (as a multiple of the gravitational acceleration g).

2. **Harmonic Oscillator**

   Consider a simple harmonic oscillator moving in one dimension with restoring force $F = -kx$. Write the Lagrangian for the harmonic oscillator and show that the Euler-Lagrange equation is Newton's second law $m\ddot{x} = -kx$.

3. **Sliding on a Cycloid**

   Consider a cycloidal track (like we found for the brachistochrone)

   $$x = a(\theta - \sin\theta)\ , \quad y = a(1 - \cos\theta) \tag{1}$$

   with y increasing downward. The generalized coordinate θ extends from $\theta = 0$ at the left of the track $x = 0, y = 0$ to $\theta = 2\pi$ at the right $x = 2\pi a, y = 0$, and the lowest point of the track is $x = a\pi, y = 2a$ at $\theta = \pi$.

   To learn more about motion on the cycloid, try a different generalized coordinate. Define

   $$s = \int_{\pi}^{\theta} d\theta' \sqrt{\left(\frac{dx}{d\theta'}\right)^2 + \left(\frac{dy}{d\theta'}\right)^2}\ , \tag{2}$$

   so $|s|$ is the distance traveled from the lowest point along the cycloid.

   (a) Find the kinetic energy $T = m(\dot{x}^2 + \dot{y}^2)/2$ in terms of s. *Hint:* Use the infinitesimal form of the Pythagorean theorem to find $\dot{s}^2$.

   (b) Write the potential energy $V = -mgy$ in terms of s. *Hint:* Using a half-angle formula, integrate (2) to get s in terms of θ. Then use the angle addition formula to get s^2 in terms of y.

(c) Using your results for the kinetic and potential energies, write the Lagrangian in terms of s. By comparing the Lagrangian to your result from problem 2, show that motion on the cycloid is simple harmonic. You do not need to find the Euler-Lagrange equations. (This actually proves that the sliding on the cycloid has a period that is independent of amplitude, just like any other harmonic oscillator.)