

PHYS-3203 Homework 10 NOT TO BE HANDED IN

Solutions will be posted 14 April 2022.

1. Masses with Unbalanced Springs *from a problem by Cline*

Consider two identical masses m and three springs arranged in a line between two walls so that each mass is attached to a wall by a spring and the third spring connects the masses. The spring constants are chosen so the Lagrangian becomes

$$L = \frac{m}{2} (\dot{x}_1^2 + \dot{x}_2^2) - \frac{k}{2} (4x_1^2 - 4x_1x_2 + 7x_2^2) , \quad (1)$$

where x_1 and x_2 are the displacements of the two masses from their equilibrium positions.

- (a) Find the equations of motion (Euler-Lagrange equations).
- (b) We can write the normal modes of the system as vectors $[x_1 \ x_2]^T = [B_1 \ B_2]^T \exp(i\omega t)$. Write the equations of motion for a normal mode as a matrix equation.
- (c) Find the normal mode frequencies.
- (d) By substitution into the Lagrangian (1), verify that $\eta_1 = \sqrt{m/5}(x_1 - 2x_2)$ and $\eta_2 = \sqrt{m/5}(2x_1 + x_2)$ are respectively the normal coordinates for the higher and lower normal mode frequencies.