

PHYS-3203 Homework 1 Due 19 Jan 2022

This homework is due to <https://uwcloud.uwinnipeg.ca/s/QGK3eGfDRgND6sC> by 10:59PM on the due date. Your file(s) must be in PDF format; they may be black-and-white scans or photographs of hardcopies (all converted to PDF), PDF prepared by LaTeX, or PDF prepared with a word processor *using an equation editor*.

1. Crossing the Line

A light ray travels through a medium with index of refraction n_1 for $x < 0$ and index n_2 for $x > 0$ starting at position $(-X, 0)$ and ending at position (X, Y) for $X \gg Y$. Use Snell's Law to show that the travel time is minimized when $y = n_2 Y / (n_1 + n_2)$. *Hint:* use $X \gg Y$ to argue that the angles of incidence and refraction are small and the fact that $\tan \theta \approx \sin \theta$ for small angles.

2. A Line Really Is Minimum Length from Thornton & Marion and others

We know that the minimum length curve in two dimensions that connects the origin to the point $x = y = a$ is the straight line $y(x) = x$. Consider instead the curve $y(x) = x + b \sin(n\pi x/a)$, which also connects the origin to $x = y = a$ if n is an integer.

- Write the length of this curve as an integral over x and, assuming $b \ll a$, expand the integrand to second order in b/a (that is, use a Taylor series for the variable $z = b/a$). *Hint:* use an infinitesimal version of the Pythagorean theorem for the length.
- From your previous result, show that the length of the curve to this order is $\sqrt{2}a + cb^2/a$, where c is a positive number, and find c . This shows that changing the line to a slightly different curve increases the path length. *Hint:* you may find an angle addition formula useful for your integral.

3. A Sample Euler-Lagrange Equation from Cline 5.2

Consider the functional

$$S = \int_{x_1}^{x_2} dx \sqrt{x} \sqrt{1 + (y')^2}, \quad (1)$$

which depends on the path $y(x)$.

- Find the Euler-Lagrange equation.
- Solve the Euler-Lagrange equation to find the path $y(x)$ that extremizes S . Show that this path satisfies $x = ay^2 + b$ for some constants a and b .