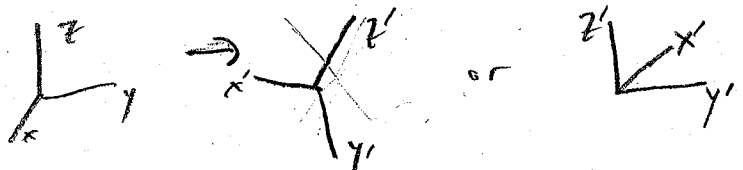
# Newton's Laws

This course starts the more advanced study of Newtonian Mechanics

- Space & Time
  - We make fundamental assumptions about space & time
    - Continuous in nature (true in quantum gravity?)
    - It's possible to use a standard scale across the universe
  - In Newtonian (classical) Mechanics, we make more assumptions
    - No limits to measurement accuracy / no fuzziness (classical vs quantum)
    - There is an absolute time / clock system
    - Geometry is Euclidean

    } (Newtonian vs Einsteinian / relativistic)

- Positions
  - Time is described with a single number $t$, which all observers agree on up to a choice of time origin
  - Spatial position is given by coordinates with respect to 3 $\perp$ axes $(x, y, z)$.
    - + Can choose the origin $(x,y,z) = (0,0,0)$
    - + Can choose orientation of axes by rotating and/or reflecting an initial set




By convention, we always follow the right-hand-rule for $(x, y, z)$

  - Positions are vectors and can be written in components $r_i = (x, y, z)$ $(i = 1, 2, 3)$ or as a sum $\vec{r} = x\hat{\imath} + y\hat{\jmath} + z\hat{k}$, where $\hat{\imath}, \hat{\jmath}, \hat{k}$ (or $\hat{x}, \hat{y}, \hat{z}$) are unit vectors along the axes.
    - + $\vec{r}$ is well-defined as a mathematical object independent of axis choice. The sum + components above are w.r.t. an axis's choice
    - +. We will review more later.

## Newton's 1st Law

Best stated as "An object remains in the same state of motion unless acted on by an outside force"

- Only true in inertial reference frames. This is a choice of time origin + spatial axes at rest w.r.t. an unaccelerated observer.
  - Circularly, an accelerated observer sees objects accelerate w/o a force to cause it.
- The Relativity Principle: Physics is the same in all inertial frames
  - Only relative velocities + positions matter, so forces between objects depend only on $\vec{r}_i - \vec{r}_j$ or $\vec{v}_i - \vec{v}_j$
  - We can choose a convenient frame to work in.
  - Means observers moving past each other at constant velocity must agree on results of any experiment.

## Newton's Third Law

"Any action has an equal but opposite reaction"

- By the relativity principle, any force on object $i$ must be due to some object $j$.
  - Denote this force on $i$ by $j$ as $\vec{F}_{ij}$
  - So for example $\vec{F}_{giv} = m_i \vec{g}$ gravitational force is really the force $\vec{F}_{i\oplus}$, the force on object $i$ by the earth $\oplus$
- The 3rd law states $\vec{F}_{ij} = -\vec{F}_{ji}$. → Note about electrodynamics
- Consider an object made of particles $i, j$. The total force on all constituents is

$$\sum_{i,j} \vec{F}_{ij} = \frac{1}{2} \sum_{i,j} (\vec{F}_{ij} + \vec{F}_{ji}) = 0$$

  - Internal forces all cancel out; they just hold the object together
  - An external force is needed to change the motion of the object

◎ Newton's 2nd Law $\vec{F}_i = m_i \vec{a}_i$ where $\vec{F}_i$ = total force on $i$ ← Gives equation of motion (EOM)

- Definitions
  - $m_i$ = inertial mass of $i$
  - Position $\vec{r}_i(t)$ has velocity $\vec{v}_i(t) \equiv \frac{d\vec{r}_i}{dt}$ and acceleration $\vec{a}_i = \frac{d\vec{v}_i}{dt}$

    We will generally use dot notation for time derivatives

    $$\vec{v} = \dot{\vec{r}}, \quad \vec{a} = \ddot{\vec{r}}$$

  - The momentum of the object is $\vec{p} = m\vec{v}$, so the 2nd law is more succinctly $\vec{F}_i = d\vec{p}_i/dt$
  - 3rd Law means momentum is conserved
- What do we mean by mass?
  - For $\vec{F} = m\vec{a}$ to make sense in context of the 3rd law, + any two colliding isolated objects must have $\vec{a}_1 = -k_{12}\vec{a}_2$.
    + This constant is an intrinsic property of the two objects. (indep. of details of the collisions)
    + If object 3 collides with object 2 collides with object 1, then $k_{13} = k_{12}k_{23}$
    + So these constants divide into the ratio of individual object properties $k_{12} = m_2/m_1$
  - If two objects are joined together, their masses add. (Consider a 3-body collision)
  - The fact that inertial mass (above) and gravitational mass (as in Newton's law of gravitation) can be chosen equal is known as the equivalence principle.

◎ Dimensional Analysis
- Make sure to check units of your answers
- Can also find functional dependence of quantities by checking units
  - Pendulum: has a length, $l$ and mass $m$ moving in grav. acc. $g$, amp. $\theta_0$. The period $T = m^\alpha l^\beta g^\gamma$. Units require $\alpha = 0$, $\beta = -\gamma$, $\gamma = 1/2$
  - Angular amplitude is dimensionless, so $T = f(\theta)\sqrt{l/g}$