PHYS-3202 Homework 7 Due 12 Nov 2020

This homework is due to `https://uwcloud.uwinnipeg.ca/s/LLijRqSDKdXgMDA` by 10:59PM on the due date. You may submit a Word doc/docx document (with an equation editor for mathematics) or a PDF (typed or black-and-white scanned).

1. **'Oumuamua** *extended from a Fowles & Cassiday problem*

   The earth's orbit is nearly circular. In this problem, assume that it is exactly circular with radius 1 AU and period 1 year. Note that 1 AU is approximately 1.5×10^8 km and 1 year is approximately 3×10^7 s.

   (a) Find the earth's orbital speed $v_\oplus$ in terms of its orbital radius $r_\oplus$ and physical constants (Newton's constant, solar mass, earth mass, etc).

   (b) An object is observed at a distance $dr_\oplus$ (that is, d AU) from the sun moving at a speed given by $fv_\oplus$. Find the condition on d and f that determines if the object has an elliptical or hyperbolic orbit. *Hint*: use the relationship you found in the previous part.

   (c) In October 2017, the PanSTARRS telescope discovered a small object later named *'Oumuamua* (the Hawaiian word for "scout"). On 10 Oct 2017, it was at a distance of 1 AU moving at a speed of about 50 km/s relative to the sun. The perihelion distance was approximately 0.25 AU. Use your result from above to determine if its orbit is elliptical or hyperbolic. If the orbit is elliptical, determine the aphelion distance; if the orbit is hyperbolic, determine the asymptotic speed relative to the sun. Give your answers to 1 significant digit.

2. **Collisional Cross Section of the Earth**

   Consider meteoroids approaching the earth with positive asymptotic speed v and impact parameter b. This means the meteoroids have positive total energy. In this problem, ignore friction due to the earth's atmosphere and acceleration around the sun.

   (a) Find the largest impact parameter b_{max} such that the meteoroid will collide with the earth. *Hint:* For a collision to happen, the perigee of the meteoroid's hyperbolic orbit must be less than the earth's radius R.

   (b) Find the total cross section σ for the asteroids to collide with the earth as a function of the asteroid's asymptotic speed.

   (c) How much larger is this cross section than the earth's geometric cross section πR^2? Give your answer as a fractional difference (ie, $(\sigma - \pi R^2)/\pi R^2$) first in terms of the earth mass and radius and the meteoroid's asymptotic speed and then as a dimensionless number for a typical meteoroid speed of $v = 20$ km/s (relative to earth). Does gravity make a significant difference in the likelihood of a meteoroid hitting the earth? You may find astronomical data at the Particle Data Group. Use the nominal equatorial radius for the earth.