PHYS-3202 Homework 6 Due 4 Nov 2020

This homework is due to `https://uwcloud.uwinnipeg.ca/s/LLijRqSDKdXgMDA` by 10:59PM on the due date. You may submit a Word doc/docx document (with an equation editor for mathematics) or a PDF (typed or black-and-white scanned).

1. **A Different Central Force** *a clarified problem by Taylor*

   The inverse square central force is very special because the orbits are closed (when the angle moves from 0 to 2π, the radius comes back to the same place). In this problem, consider a particle of mass m moving in a central force $\vec{F} = -(k/r^{5/2})\hat{r}$, where k is a positive constant. We will plot an orbit and see that it is not closed. You will need to use Maple or Mathematica; attach a PDF copy of your code to your assignment. Label which parts of the code correspond to the different parts of the problem.

   (a) What is the potential energy $V(r)$ for this force?

   (b) Write the effective potential for the particle if the angular momentum is $\vec{J}$. Find the radius r_0 of a circular orbit. In units where m, k, and J are all 1 (we have to choose mass, length, and time units to do this), use Maple/Mathematica to plot the effective potential over the range $0 < r < 5r_0$ with vertical range $-0.5 < U < 1$. By eye, this should not look much different than the usual inverse square effective potential. *Maple:* Either use the `view` option or the axis properties menu from right-clicking the plot to set the vertical range. *Mathematica:* Use the `PlotRange` option.

   (c) Suppose the particle has energy $E = -0.1$ in the units used for your plot in part (b). Find the minimum and maximum radius that the particle can reach. Give your answers to 3 significant digits. *Hint:* Use Maple's `fsolve` or Mathematica's `FindRoot` function; you may need to set the starting guess for these functions to different values two find the two values of the radius.

   (d) Following our discussion in the lecture notes for the inverse square law, write the conserved energy in terms of the variable $u = 1/r$ and its derivative with respect to ϕ. Then take the derivative of this expression with respect to ϕ to find a second order equation for u as a function of ϕ. Leave m, k, J as variables in this part.

   (e) Find a numerical solution $u(\phi)$ for the second order differential equation from part (d) (use the units of part (b)). Use initial conditions $u(0) = 1/r_{min}$, $du/d\phi(0) = 0$, where r_{min} is the minimum radius you found in part (c). Then plot the orbit in the $xy-$plane as a parametric plot of x and y versus ϕ (note that $x = \cos(\phi)/u(\phi), y = \sin(\phi)/u(\phi)$) for $\phi = 0$ to 8π. Does the orbit appear to be closed (in other words, does the orbit return to the initial radius value for $\phi = 2\pi n$ with n an integer)? *Hint:* The code to solve the differential equation and make the parametric plot should use the same commands as in assignment 5 in either Maple or Mathematica.

2. **Energy of an Elliptical Orbit**

   Show that the total energy of an elliptical orbit is $E = k/2a$, where a is the semi-major axis and k is the (negative) constant in the potential energy $V = k/r$.

3. **Hohmann Transfer** *adapted from Kibble & Berkshire and others*

   Suppose we want to send a space probe from earth to a planet farther from the sun. The most fuel-efficient orbit for the probe is known as a *Hohmann transfer*. Its perihelion is at earth's orbit, and its aphelion is at the other planet, which we will choose to be Jupiter. Assume that both earth and Jupiter have circular orbits (the eccentricities are less than 0.05 in both cases) with semi-major axes $a_\oplus$ and a_J respectively.

(a) Find the required semi-major axis and eccentricity of the transfer orbit for the spaceprobe.

(b) What is the transit time for the orbit? Give your answer first in terms of $a_\oplus, a_J$ and then in years given that $a_J \approx 5$ AU. Note that the transit time is half the period of the orbit.