PHYS-3202 Homework 5 Due 28 Oct 2020

This homework is due to `https://uwcloud.uwinnipeg.ca/s/LLijRqSDKdXgMDA` by 10:59PM on the due date. You may submit a Word doc/docx document (with an equation editor for mathematics) or a PDF (typed or black-and-white scanned).

1. **Multiple-Choice from a Previous Exam** *from MIT OpenCourseWare*

   For each part, choose the correct answer from the options given and explain your answer in no more than two lines. *On the upcoming term test, you will not need to explain your answer for the multiple choice. These questions are slightly modified from a previous term test.*

   (a) A triathlete rides a bike up a hill. In what direction does the friction from the ground act on the bike? The wheels of the bike do not slip on the ground.
   A. $F_{friction} = 0$ B. down the hill C. up the hill D. perpendicular to the ground

   (b) A hockey puck of mass m and velocity $\vec{v}$ strikes a stationary puck of mass $3m$ elastically. Which of the following could be the final velocities of the two pucks?
   A. $-\vec{v}/2$ and $\vec{v}/2$ B. $\vec{v}/2$ and $3\vec{v}/2$ C. $-3\vec{v}$ and $\vec{v}$ D. 0 and $\vec{v}/3$

2. **Isotropic Oscillator with Damping**

   Consider a three-dimensional isotropic harmonic oscillator with spring constant $k = m\omega_0^2$ that also experiences an isotropic linear damping force $\vec{F}_{damp} = -2m\gamma\vec{v}$. Assume $\gamma \ll \omega_0$.

   (a) The damping force also provides a torque on the oscillator. Show that this torque is always proportional to the angular momentum. Use this fact to show that the angular momentum has exponential time dependence and further that motion remains in a plane.

   (b) Find the general solution for the x position of the oscillator as a real function of time (ie, in a form that is manifestly a real number). Note that the general solution for y is the same but with different integration constants due to different initial conditions.

   (c) Use your general solution from part (b) to find the position as a function of time given initial conditions $\vec{r}(0) = x_0\hat{\imath}$ and $\vec{v}(0) = v_0\hat{\jmath}$. *Hint:* Be careful about time derivatives.

   (d) Find the angular momentum of your solution from part (c) and confirm that it has the correct exponential time dependence that you found in part (a).

3. **An Extended Isotropic Spring** *from Kibble & Berkshire*

   One end of a spring is attached to the origin, and the other is attached to a mass m. The spring can swivel freely in two dimensions (horizontal, so we neglect gravity). The equilibrium length of the spring is a, so the restoring force on the mass is $-k(\rho - a)$ in cylindrical coordinates. Ignore damping. Note that cylindrical coordinates in the xy plane are the same as spherical polar coordinates in the equatorial plane with some renaming.

   (a) Suppose the mass oscillates linearly (ie, in the x direction with $y = 0$ at all times). What is the angular frequency ω_0 of oscillation?

   (b) For motion with angular momentum $\vec{J} = J\hat{z}$, find the effective potential for radial motion.

   (c) Suppose the mass has initial conditions $\rho = a, \varphi = 0$ and $\dot{\rho} = 0, \dot{\varphi} = \omega$. If the mass reaches a maximum radius of $2a$, find ω and the value of $\dot{\varphi}$ when $\rho = 2a$. Write your answers as multiples of the natural frequency ω_0.

(d) Now suppose that the mass is moving in a circular orbit of radius $\rho = 2a$. Find the angular velocity of the orbit (which is also the frequency of the orbit) in terms of the natural oscillator frequency ω_0. *Hint:* Where is the circular orbit in terms of the effective potential for radial motion?

(e) The mass is in the circular orbit described in part (d). At time $t = 0$, it experiences a sharp radially directed force, so it gains a small component of velocity in the $\hat{\rho}$ direction, which leaves the angular momentum unchanged. The mass subsequently oscillates around the circular orbit. Find the frequency ω' of this oscillation in terms of ω_0 and give a qualitative description of the mass's motion (you will need the answer from part (d) also). *Hint:* Remember that the circular orbit is the minimum of the effective potential energy and that a potential energy function near its minimum always looks like a harmonic oscillator potential energy.

4. **Projectile Motion with Linear and Quadratic Air Resistance**

We have been able to give exact expressions for position as a function of time for projectile motion with no air resistance and linear air resistance. However, that is not possible for other types of air resistance, so we have to solve Newton's laws using numerical techniques. This problem will explore numerical solutions for ballistic motion for no air resistance, linear air resistance, and quadratic air resistance.

Instructions for Computational Solutions: You may use either Maple (through UWinnipeg Citrix Receiver) or Mathematica Online for parts (b) through (e). You should submit a PDF copy of your code either as part of your solution document or an additional file (example file name for me: AFrey_hw5_q4.pdf). Example files for both Maple and Mathematica are available for the solution to part (b). Detailed instructions on how to use either of these software packages and submit your work are given here under the Assignment Policies section (just above homework assignments) on the course web page.

(a) The air resistance force is $\vec{F} = -mA\vec{v}$ for linear air resistance and $\vec{F} = -mBv\vec{v}$ for quadratic air resistance, where $\vec{v}$ is the objects velocity and v its speed. Since ballistic motion is confined to the $(x, z)-$plane, show that Newton's second law for x and z in each case can be written as

$$\ddot{x} = 0 \quad , \quad \ddot{z} + g = 0 \quad \text{(no air resistance)} \tag{1}$$

$$\ddot{x} + A\dot{x} = 0 \quad , \quad \ddot{z} + A\dot{z} + g = 0 \quad \text{(linear)} \tag{2}$$

$$\ddot{x} + B\dot{x}\sqrt{\dot{x}^2 + \dot{z}^2} = 0 \quad , \quad \ddot{z} + B\dot{z}\ \sqrt{\dot{x}^2 + \dot{z}^2} + g = 0 \quad \text{(quadratic)} \tag{3}$$

(b) Find a numerical solution to the differential equations (1) representing projectile motion without air resistance; treat them as coupled differential equations. The projectile is initially at the origin with speed 45 m/s at an angle 0.4 radians from the horizontal. Then use a numerical equation solver to find the time when $z = 0$ and plug that time back into the x position to find the range (the distance the projectile travels horizontally before hitting the ground). *Hint*: The sample Maple and Mathematica files on the web page will do this for you. You just need to run them and use them as a template for the following parts of the problem.

(c) Repeat part (b) for the linear air resistance case described by equations (2). Use $A = 0.1$ 1/s.

(d) Repeat part (b) for the quadratic air resistance case described by equations (3). Use $B = 0.01$ 1/m.

(e) Plot the three trajectories you found in parts (b,c,d) as functions z vs x on the same plot. In Maple, use the `with(plots):` command, so the `plot` command gains the ability to make parametric plots (you can make all three with a single command or use the `display` command to combine separate plots). In Mathematica, you will want to make a parametric plot for each plot then use the `Show` command to combine the plots. Use a different color for each plot. *Hint*: You can find how to do parametric plots by googling "maple parametric plot" or "mathematica parametric plot." Look at plot options for how to change colors.