PHYS-3202 Homework 4 Due 7 Oct 2020

This homework is due to `https://uwcloud.uwinnipeg.ca/s/LLijRqSDKdXgMDA` by 10:59PM on the due date. You may submit a Word doc/docx document (with an equation editor for mathematics) or a PDF (typed or black-and-white scanned).

1. **Impulse on a Harmonic Oscillator** *loosely inspired by Fowles & Cassiday*

   Consider an undamped ($\gamma = 0$) harmonic oscillator with natural frequency ω_0. Suppose that it experiences a driving force $F(t) = F\exp(-\alpha t)$ (with F, α real) and satisfies the initial conditions $x(0) = 0, \dot{x}(0) = 0$.

   (a) Show that the solution at very long times can be written as

   $$x(t) = -\frac{(F/m\omega_0)}{\sqrt{\alpha^2+\omega_0^2}}\cos(\omega_0 t + \theta)\ ,\ \text{where}\ \tan\theta = \alpha/\omega_0\ . \tag{1}$$

   Hint: Find the homogeneous and particular solutions and note which ones last to long times. Also look for how the angle θ can appear.

   (b) The *impulse* of a force on an object is the momentum imparted to the object by the force, which is the integral of the force over time. Show that the impulse on the oscillator from the driving force after time $t = 0$ is $\Delta p = F/\alpha$. Now consider the limit $\alpha \to \infty$ with Δp a fixed number, which is the limit that the force acts very quickly. Find the solution (1) in this limit.

   (c) We can also think of a quick impulse Δp on a harmonic oscillator (which starts at its equilibrium point) as setting initial conditions $x(0) = 0, \dot{x}(0) = \Delta p/m$. Write the solution for the harmonic oscillator with these initial conditions but no driving force. Is this the same as your result from the previous part?

2. **A Couple of Vector Identities** *from Kibble & Berkshire*

   (a) Using vector triple-product identities, write $(\vec{a}\times\vec{b})\cdot(\vec{c}\times\vec{d})$ in terms of the dot products $\vec{a}\cdot\vec{c}$, $\vec{b}\cdot\vec{c}$, $\vec{a}\cdot\vec{d}$, and $\vec{b}\cdot\vec{d}$

   (b) Verify the identity $\vec{\nabla}\times(\vec{a}\times\vec{b}) = (\vec{\nabla}\cdot\vec{b})\vec{a} + (\vec{b}\cdot\vec{\nabla})\vec{a} - (\vec{\nabla}\cdot\vec{a})\vec{b} - (\vec{a}\cdot\vec{\nabla})\vec{b}$ by comparing the z component of each side of the identity.

3. **Spherical Polar Unit Vectors**

   In this problem, you will find the components of the spherical polar unit vectors $\hat{r}, \hat{\theta}, \hat{\phi}$ in terms of the Cartesian unit vectors $\hat{\imath}, \hat{\jmath}, \hat{k}$.

   (a) We know that the velocity of an object is given by $\vec{v} = \dot{x}\hat{\imath} + \dot{y}\hat{\jmath} + \dot{z}\hat{k}$. Begin by finding expressions for $\dot{x}$, $\dot{y}$, and $\dot{z}$ in terms of spherical polar coordinates r, θ, ϕ and their time derivatives.

   (b) By comparing to the form of $\vec{v}$ in spherical coordinates, find $\hat{r}, \hat{\theta}, \hat{\phi}$.

   (c) Use your results from the previous two parts to find the acceleration vector in spherical polar components (that is, as components multiplying $\hat{r}, \hat{\theta}, \hat{\phi}$). *Hint:* $\hat{\imath}, \hat{\jmath}, \hat{k}$ are time-independent.