PHYS-3202 Homework 3 Due 30 Sept 2020

This homework is due to `https://uwcloud.uwinnipeg.ca/s/LLijRqSDKdXgMDA` by 10:59PM on the due date. You may submit a Word doc/docx document (with an equation editor for mathematics) or a PDF (typed or black-and-white scanned).

1. **Hanging Spring**

   Consider a mass m on a spring with potential energy $kx^2/2$, where $x = 0$ is the equilibrium extension. Suppose the spring is hung from the ceiling (with x increasing downwards).

   (a) Write the potential energy as a function of x with the inclusion of gravity and find the new equilibrium point x_0.

   (b) Rewrite the potential in terms of $y = x - x_0$. From the form of the potential only, argue that the motion of the hanging spring is harmonic oscillation around $y = 0$ and find the frequency of oscillation. Do not solve any differential equations.

2. **Work Done on a Forced Oscillator** *from Kibble & Berkshire 2.25*

   Consider a harmonic oscillator with damping γ and natural frequency ω_0 that experiences a force $F(t) = F\cos(\omega t)$.

   (a) In class, we found the particular solution for x when the driving force is a complex exponential $F(t) = F\exp(i\omega t)$. If that complex solution is $x_1(t)$, show that $x(t) = (x_1(t)+x_1^*(t))/2$ is a solution for the cosine driving force in this problem. Find $x(t)$ for initial conditions $x(0) = 0, \dot{x}(0) = 0$ assuming $\omega_0 > \gamma$.

   (b) Recall that the power, or work done on an object per unit time, is $P = F\dot{x}$. Find the power on the oscillator by the force $F(t)$ at time t for the steady-state solution (ie, late time solution neglecting transients). Then find its average over one period.

   (c) The damping force is $-2m\gamma\dot{x}$. Find the power by the damping force at time t and the average power over one period for the steady-state solution. Show that your result is the opposite of the power from the driving force, so the total work done on the oscillator over a period is zero.

3. **Damped Unstable Equilibrium**

   Consider movement around the top of a parabolic potential including damping and an external driving force. The equation of motion is

   $$\ddot{x} + 2\gamma\dot{x} - \kappa^2 x = F(t)/m \ , \qquad (1)$$

   where γ and κ are positive constants.

   In the first 2 parts, set the driving force $F(t) = 0$.

   (a) Find the general solution to (1). Show that x grows exponentially in t after a short period of time.

   (b) Assume x is given by the exponentially growing solution only. Show that the acceleration term in (1) is negligible when $\kappa \ll \gamma$. In other words, show that $|\ddot{x}| \ll 2\gamma|\dot{x}|$ and $|\ddot{x}| \ll \kappa^2|x|$. Physics very similar to this is important in the theory of inflation, which postulates that the early universe expanded very rapidly. *Hint:* for each inequality, take the ratio of the smaller to larger side and expand in terms of the small number κ/γ.

(c) Now consider sinusoidal forcing $F(t) = Fe^{i\omega t}$. Write the general solution, including the solutions to the non-driven equation. Is it possible to neglect these extra terms after sufficient time passes?