PHYS-3202 Homework 2 Due 23 Sept 2020

This homework is due to `https://uwcloud.uwinnipeg.ca/s/LLijRqSDKdXgMDA` by 10:59PM on the due date. You may submit a Word doc/docx document (with an equation editor for mathematics) or a PDF (typed or black-and-white scanned).

1. **Turbulent Air Resistance**

   Consider an object falling in a uniform gravitational acceleration g against a drag force of magnitude λv^2. In this problem, you will want to recall the hyperbolic trig functions and the relationships $\cosh^2\theta - \sinh^2\theta = 1$, $d\cosh\theta/d\theta = \sinh\theta$, and $d\sinh\theta/d\theta = \cosh\theta$.

   (a) Show that the speed of the object as a function of time is

   $$v(t) = \sqrt{\frac{mg}{\lambda}}\tanh\left(\sqrt{\frac{\lambda g}{m}}\,t\right) , \qquad (1)$$

   where m is the object's mass. Assume that $v = 0$ at $t = 0$. Does this formula agree with the terminal velocity from the lecture notes? *Hint:* You can directly integrate Newton's 2nd law.

   (b) Now find the distance traveled as a function of time. Check that your answer has the correct units.

2. **Bouncing Ball** *inspired by Kibble & Berkshire 2.28*

   A ball is released from rest at height h and bounces off the floor with coefficient of restitution e for each bounce. Treat its motion as entirely one-dimensional.

   (a) Show that the ball comes to rest on the floor at time

   $$t = \frac{1+e}{1-e}\sqrt{\frac{2h}{g}} \qquad (2)$$

   (including the time before the first bounce).

   (b) Find the total distance that the ball travels including the distance before the first bounce.

3. **Sample Inelastic Collision**

   Two carts of mass m_1 and m_2 initially move at velocities u_1 and u_2 along a frictionless linear track. The carts collide with coefficient of restitution e. Find the final velocities v_1 and v_2 (with sign). If $u_2 = 0$ and $v_1 = fu_1$ (where $-1 < f < 1$), what is v_2?

4. **Rocket Science**

   Consider a rocket of initial velocity v_0 and initial total mass (including fuel) m_0 moving linearly in outer space. Recall from class that its velocity at a later time t is $v = v_0 + u\ln(m_0/m)$, where u is the exhaust speed relative to the rocket and m is the mass at time t.

   (a) *from Thornton & Marion* What is the ratio m/m_0 when the momentum of the rocket is maximized? *Hint:* Remember that the mass of the rocket is changing as it burns and exhausts fuel.

   (b) *from Cline 2.10* Assume the rocket exhausts fuel at a constant rate $\dot{m} = -k$ (until the fuel runs out). Find the displacement as a function of time.

   (c) Finally, assuming that the relative exhaust speed u is constant, what is the mass of the rocket as a function of time if its acceleration a is constant?