PHYS-3202 Homework 11 NOT DUE

This homework is for your study purposes only and is not to be turned in. These are a bit more involved than you should expect on the exam but illustrate the concepts.

1. **Air Friction** *from Fowles & Cassiday*

   Consider a disc-shaped symmetric object with $I_1 = I_2 \equiv I < I_3$, which is appropriate for a flat cylinder. While spinning in the air, it experiences a drag-like torque $\vec{\tau} = -k\vec{\omega}$ known as air friction. The disc initially has angular velocity with $\omega_3 \gg \omega_1, \omega_2$ in terms of the components along the principal axes. *Hint:* this type of torque is one that can be analyzed easily using Euler's equations.

   (a) Show that the angular velocity around the symmetry axis $\hat{e}_3$ decreases exponentially in time.

   (b) Next, show that the angle between $\vec{\omega}$ and the symmetry axis decreases in time. That is, $(\omega_1^2 + \omega_2^2)^{1/2}$ decreases more rapidly in time than ω_3.

   (c) Finally, argue that $\vec{\omega}$ goes to a fixed angle in the $\hat{e}_1, \hat{e}_2$ plane as $t \to \infty$. You may use your solution to the previous part.

2. **Wobbling Space Station** *inspired by Fowles & Cassiday*

   A space station in the form of a uniform ring of radius R, negligible thickness and height, and mass M is initially rotating around its symmetry axis $\hat{e}_3$ with angular velocity ω_0 ($\vec{\omega} = \omega_0 \hat{e}_3$). Its principal moments are $I \equiv I_1 = I_2 = MR^2/2$, $I_3 = MR^2$.

   (a) A meteoroid strikes the space station at its edge, imparting a momentum of magnitude p parallel to the symmetry axis. What are the frequency and amplitude of the precession of $\vec{\omega}$ around the symmetry axis after the collision? (This is what residents of the space station feel as oscillations in the "artificial gravity.")

   (b) To try to fix the problem, the space station turns on thrusters at the edges of the space station on the $\pm\hat{e}_2$ axes that produce a constant thrust $F/2$ each in the $\pm\hat{e}_3$ direction. Show that turning on the thrusters with force $F = p\omega_0/2$ when $\omega_1 = 0, \omega_2 > 0$ and leaving them on for a time $t = \pi/\omega_0$ will stop the precession of the station.

   Hint: In both parts, you will have a pair of differential equations in 2 variables. Differentiate one and eliminate a variable to find a solution.