PHYS-3202 Homework 10 Due 7 Dec 2020

This homework is due to `https://uwcloud.uwinnipeg.ca/s/LLijRqSDKdXgMDA` by 10:59PM on the due date. You may submit a Word doc/docx document (with an equation editor for mathematics) or a PDF (typed or black-and-white scanned).

1. **Inertia Tensor of a Thin Plate**

   Consider a thin plate of material, which is effectively two-dimensional (this is known as a *lamina*). In this problem, assume that the lamina lies in the $z = 0$ plane.

   (a) *from Taylor* Prove that the moments of inertia $I_{zz} = I_{xx} + I_{yy}$ and the products of inertia $I_{xz} = I_{yz} = 0$.

   In the rest of the problem, assume that the lamina is a rectangle of sides with length $2a$ parallel to the x axis and length a parallel to the y axis. The center of the rectangle is at the origin. The lamina has uniform mass surface density and total mass M. *based on questions from FC and KB*

   (b) Calculate the inertia tensor around this origin given these axes. You may use the results of part (a) to simplify your calculations. (This inertia tensor applies to rotation around any axis through the center of the lamina.) What are the principal axes?

   (c) Use the parallel axis theorem to find the inertia tensor of the lamina around the corner located at $x = -a, y = -a/2$ (so that the x and y axes lie along the sides and z is still perpendicular).

   (d) If the lamina rotates around the long edge through this corner with constant frequency ω, what is the angular momentum as a function of the angle between the lamina and the xy plane? What torque must the axis exert on the lamina as a function of that angle?

2. **Finding Principal Axes**

   Four identical small balls of mass m each are at the following locations in the xy plane: $(x, y, z) = (a, 0, 0), (-a, 0, 0), (a/\sqrt{3}, 2a/\sqrt{3}, 0), (-a/\sqrt{3}, -2a/\sqrt{3}, 0)$. They are held together by very light rods. Treat the balls as idealized point particles and the rods as massless.

   (a) Find the (3D) inertia tensor of this object around the origin. You may use your result from question 1(a).

   (b) Find the principal axes and corresponding moments of inertia for the object.

3. **Kater's reversible pendulum** *based on a problem from Thornton & Marion and others*

   Consider an object hung from a pivot a distance L from the center of mass, which is a physical pendulum with frequency $\omega = \sqrt{MgL/I}$, where I is the moment of inertia around the pivot, as we've seen. Suppose we can flip the pendulum over and hang it from a pivot a distance L' on the other side of the center of mass (with a parallel axis of rotation). If the pendulum has the same frequency of oscillation around this second pivot, show that $MLL' = I^{CM}$, the moment of inertia for the parallel axis through the center of mass.

4. **Start Up of Rolling** *a problem seen many places*

   A thin hoop of mass M and radius R is spinning around the axis through its center with the axis held horizontally. The initial angular velocity is ω_0 when the hoop is placed on a surface with coefficient of kinetic friction μ_k (at time $t = 0$). When does the hoop stop slipping (ie, begin rolling without slipping)? How far has it traveled since then?

5. **Flipping Box** *a clarified problem by Kibble & Berkshire*

Consider a uniform cubic box of mass M and side $2a$ sliding frictionlessly on a surface with velocity $\vec{v} = v\hat{\imath}$. The sides of the cube are oriented in the $\hat{\imath}, \hat{\jmath}, \hat{k}$ directions. The cube hits a barrier at $x = 0$ (extending in y), which stops the front edge instantaneously. Assume that the barrier has negligible height.

(a) Find the angular momentum of the sliding box before it collides with the barrier.

(b) What is the angular velocity of the box around the barrier immediately after the collision? *Hint:* use conservation of angular momentum around the barrier.

(c) The collision is inelastic, but energy is conserved after the collision. What is the minimum initial speed the cube needs to flip all the way over after the collision? Assume that the edge of the cube that first hits the barrier stays at $x = 0$ afterwards. *Hint:* note that the center of mass must, at a minimum, be directly over the edge if the cube flips over under the given assumption.