Intermediate Mechanics PHYS-3202 Final Exam

Dr. Andrew Frey

10-11 Dec 2020

Instructions

- This test will be available at noon CST Thurs 10 Dec 2020 and is due at noon CST Fri 11 Dec 2020
- Upload your solutions to `https://uwcloud.uwinnipeg.ca/s/LLijRqSDKdXgMDA` . **This is the same link as for homework.**
- Submissions should be PDF files that are either scanned hardcopies or prepared with LaTeX or else MS Word with an equation editor for mathematics (please export your Word file to PDF to submit). Label your filenames with your first initial, last name, and "exam" (for example `AFrey_exam.pdf`); if you need to break your assignment into multiple parts, label them in order with page numbers (`AFrey_exam1.pdf`, `AFrey_exam2.pdf`, etc). See the homework submission instructions on the course outline.
- You may consult the two course textbooks (by Idema and by Cline), lecture notes, and homework solutions linked on the course web page. You may also use Maple or Mathematica if you submit your work with questions and parts labeled as a PDF. **No other resources are allowed (including calculators, other mathematical software, etc)**. Note that using other resources or consulting other people, including other students, will be considered cheating and may lead to discipline under the University's Academic Misconduct policy and procedures.
- If you have questions, you may email me, and I will answer as soon as possible. Alternately, I will be available on zoom from 9AM to noon on 11 Dec at the usual zoom meeting for the class lectures (with short breaks) for you to ask questions and confirm that I have received your exam solutions.
- This test has 3 pages of questions (4 total pages including cover sheets).
- **Answer all questions briefly and completely.** You may re-use results in multiple problems if helpful, but please reference the first problem where you use them.

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Recent Topics: Answer all questions briefly but completely.

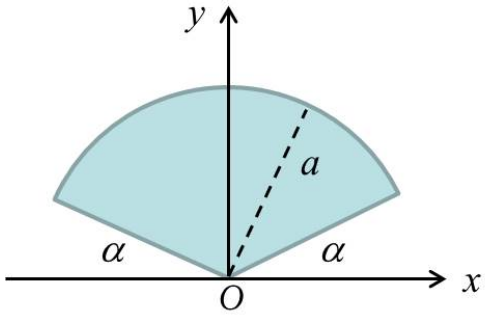
1. For each part, choose the correct answer and explain your answer *in no more than two lines.*
   (a) [5 points] A propeller-driven airplane is flying horizontally with the motor running such that the plane's angular momentum points in the forward direction. The pilot turns a rudder so the air exerts a force on the rear of the plane. In what direction must the force act on the airplane to cause the front of the airplane to tilt upward?
   A. Up B. Down C. Left D. Right
   (b) [5 points] I have two dumbbells that are identical except for their colors. I push the red dumbbell at its center of mass, and I push the blue one at one end with the same force (oriented perpendicular to the direction to the center of mass) for the same length of time. Which one has the greater kinetic energy after I am done pushing?
   A. Red B. Blue C. They are the same D. Insufficient information given to tell

2. [5 points] The hyperloop is a proposed high-speed transit system in which a pod travels through a sealed tube at approximately 1200 km/hr along the surface of the earth. Suppose two hyperloop tubes meet in Los Angeles. Is the Coriolis force greater on the hyperloop pods that travel in a tube from south to north or on the pods that travel from west to east? Assume that the pods move at the same speed.

3. [10 points] A rigid object has principal moments I_1, I_2, and $I_3 = I_1 + I_2$. The components of the angular velocity along the corresponding principal axes are ω_1, ω_2, and ω_3 respectively. Show that $\omega_1^2 + \omega_2^2$ is constant.

4. Genzel and Ghez split half the 2020 Nobel Prize in Physics for observations of the star S2 in the center of our galaxy. Star S2 orbits an object of mass M called Sgr A* with a period of 16 years, semi-major axis of 1000 AU, and pericenter distance of 120 AU. Give your answers to the following questions to 1 significant digit. Assume that S2's orbit obeys Newton's law of gravitation.
   (a) [5 points] What is the eccentricity of S2's orbit?
   (b) [10 points] Find $M/M_\odot$, where $M_\odot$ is the mass of our sun. (You should find a large mass, which indicates that Sgr A* is a black hole.)

5. The lamina in the figure below is a section of a circle with radius a and mass M. The angle α measured from the x axis to the lamina is the same on either side of the object, and the lamina lies entirely in the xy plane. The z axis points out of the page, and the lamina has uniform surface density.



Hint: Use symmetry arguments to avoid calculating integrals when possible, and carry out integrals in plane polar coordinates when necessary. You may also want to use angle addition formulae.

(a) [5 points] Find the x, y, z coordinates of the center of mass of the lamina.

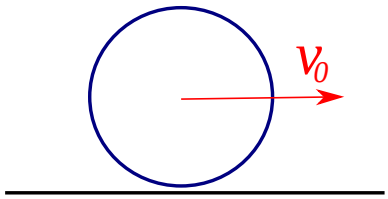
(b) [10 points] Find the components of the inertia tensor around the origin (O), including the moments and products of inertia.

(c) [5 points] What are the principal axes for rotation of the lamina *around its center of mass*? Explain your answer, but do not do a calculation.

Cumulative: Answer all questions briefly but completely.

6. [10 points] Consider a harmonic oscillator with natural frequency ω_0 and damping force $-2m\gamma\dot{x}$ forced at a frequency ω. Enough time has passed that the transients are negligible. Show that the time average of the oscillator's kinetic energy over one period is the same for $\omega = 2\omega_0$ and $\omega = \omega_0/2$.

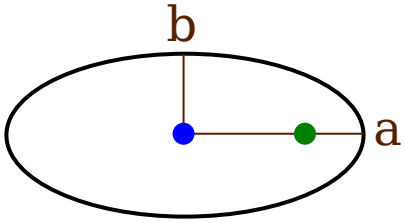
7. A thin circular hoop of mass M and radius R is initially oriented in the xy plane and moving at speed v_0 in the $+x$ direction just above the horizontal floor without rotating, as in the figure below. At time $t = 0$, the hoop contacts the floor and starts rotating due to kinetic friction. The coefficient of kinetic friction between the hoop and floor is μ_k. The hoop's moment of inertia through its symmetry axis is MR^2.



(a) [10 points] What is the speed of the hoop when it starts rolling without slipping?

(b) [5 points] While the hoop is slipping, its center of mass travels a distance x, while a point on its circumference rotates a distance θR around the hoop (counting only the rotation). The difference $x - \theta R$ is the distance that the contact point on the hoop slides past the ground. Use an argument based on kinetic energy and work to find $x - \theta R$.

8. Two particles of mass m move in elliptical orbits which are both identical to the ellipse in the figure below. The semi-major axis a and semi-minor axis b are shown as orthogonal brown lines and labeled at their endpoints with their lengths. Particle 1 is acted on by an isotropic harmonic oscillator force directed toward the center of the ellipse, indicated by the blue dot in the figure. Particle 2 is acted on by an inverse square force directed toward the focus of the ellipse shown by the green dot in the figure. Both orbits have the same period T.



(a) [5 points] What is the ratio of the speed v_1 of particle 1 at the point on the semi-minor axis labeled b in the figure to the speed v_2 of particle 2 at the same point in the figure?

(b) [10 points] Both particles start from the point labeled a on the semi-major axis at the same time. Particle 1 takes longer to reach the point b on the semi-minor axis. How much longer does particle 1 take to reach point b as a fraction of the total orbital period T?