Intermediate Mechanics PHYS-3202 Term Test

Dr. Andrew Frey

20-21 Oct 2020

Instructions

- This test will be available at 10:30AM CDT Tues 20 Oct 2020 and is due 10:29AM CDT Weds 21 Oct 2020
- Upload your solutions to `https://uwcloud.uwinnipeg.ca/s/LLijRqSDKdXgMDA` . **This is the same link as for homework.**
- Submissions should be PDF files that are either scanned (*not photographed*) hardcopies or prepared with LaTeX or else MS Word doc or docx with an equation editor for mathematics. Label your filenames with your first initial, last name, and "test" (for example `AFrey_test.pdf`); if you need to break your assignment into multiple parts, label them in order with page numbers (`AFrey_test1.pdf`, `AFrey_test2.pdf`, etc). See the homework submission instructions on the course outline.
- You may consult the two textbooks (by Idema and by Cline), lecture notes, and homework solutions linked on the course web page **but no other resources (including calculators, mathematical software, etc)**. You may also email me with questions, and I will answer as soon as possible. Note that using other resources or consulting other people, including other students, will be considered cheating and may lead to discipline under the University's Academic Misconduct policy and procedures.
- This test has 2 pages of questions (3 total pages including cover sheets). One question splits over the page break.
- **Answer all questions briefly and completely.**

PHYS-3202 Term Test 20-21 Oct 2020

Answer all questions briefly but completely. You may re-use results in multiple problems if helpful, but please reference the first problem where you use them.

1. A rocket re-enters the earth's atmosphere (moving downward) and aims its exhaust in the direction of motion in order to slow down. That is, in one-dimensional motion, the rocket velocity v and exhaust velocity are both in the positive direction.

   (a) [5 points] Show that the thrust is $+\dot{m}u$, where u is the exhaust speed and $\dot{m}$ is the rate of change of the rocket's mass. Note that this is the same as the usual thrust with $u \to -u$

   (b) [10 points] In addition to the thrust, the rocket experiences a gravitational force $F_g = mg$ in the direction of motion. Suppose it also experiences a linear air resistance $F_{drag} = -\lambda v$ (where λ is a constant) and constant mass loss $\dot{m} = -k$. Show that Newton's second law is satisfied if the velocity as a function of time is

   $$v = \frac{m_0 g}{k-\lambda}\left[\left(\frac{m}{m_0}\right)^{\lambda/k} - \frac{m}{m_0}\right] - \frac{ku}{\lambda}\left[1 - \left(\frac{m}{m_0}\right)^{\lambda/k}\right] + v_0\left(\frac{m}{m_0}\right)^{\lambda/k}, \qquad (1)$$

   where m_0 and v_0 are the mass and velocity of the rocket at $t = 0$.

   (c) [5 points] Suppose $v_0 = 0$ and $\lambda \gg k$. Find the approximate value of u for which $v = 0$ at some specific rocket mass m ($m \ll m_0$).

2. An idealized pendulum consists of a mass m at the end of a rod of length L hanging from a pivot point at the other end. The mass is free to move on a circle of radius L around the pivot; the position is measured by the angle θ from the downward vertical.

   (a) [10 points] Find the angular momentum of the pendulum bob and the torque due to gravity. Assuming that there is also a damping torque $\tau_d = -2mL^2\gamma\dot{\theta}$, write the equation of motion for the pendulum from the angular momentum equation. Show that it becomes the same as the harmonic oscillator equation for small θ. Finally, find the kinetic energy in terms of $\dot{\theta}$.

   (b) [10 points] At time $t = 0$, the pendulum is in the downward position and has kinetic energy E_0 (that is, it is starting to swing upward). Show that the energy lost by the next time the pendulum returns to the downward position is approximately $\Delta E \approx 2\pi\gamma E_0/\omega_0$ for small damping. *Hint:* Assume the angular displacement stays small and note that you can expand for small γ.

   (c) [10 points] In a grandfather clock, a pendulum with small damping keeps track of time. To keep the pendulum's amplitude constant, the clock exerts an impulse on the pendulum that increases the pendulum bob's momentum by Δp every time the pendulum passes $\theta = 0$. What is the impulse Δp required to keep the pendulum of the previous part at constant amplitude? *Hint:* Write the kinetic energy in terms of momentum p. You may approximate the impulse as being small.

3. A particle of charge q experiences the *Lorentz force* $\vec{F} = q\vec{v} \times \vec{B}$ in a magnetic field $\vec{B}$. In this problem, consider motion in a uniform magnetic field $\vec{B} = B\hat{k}$.

   (a) [5 points] Write the equations of motion for the x, y, and z coordinates of the particle and show that $\dot{z}$ is constant.

(b) [10 points] Define $u = x + iy$ and combine the equations of motion for x and y into a differential equation for u. Use that differential equation to solve for x and y as functions of time. Take the intial condition $\dot{x}(0) = 0$.

(c) [5 points] Use a choice of origin to set the average position in the xy plane equal to zero. Then confirm that the motion is circular in the xy plane by checking that $x^2 + y^2 = R^2$ at all times for some value R and that the magnitude of acceleration has the centripetal value v^2/R.