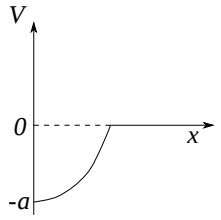
PHYS-4602 Homework 9 Due 29 March 2021

This homework is due to `https://uwcloud.uwinnipeg.ca/s/ptx3smosp2xFtmE` by 10:59PM on the due date. You may submit a PDF either scanned from handwriting or generated with LaTeX or a word processor (with an equation editor).

1. **Quadratic Well**

   Consider a particle moving in the potential

$$V(x) = \begin{cases} \infty & x < 0 \\ (m\omega^2/2)(x^2 - a^2) & 0 < x < a \\ 0 & x > a \end{cases} \qquad (1)$$



   (shown in the figure on the right).

   (a) Use the WKB approximation to estimate the bound state ($E < 0$) energies. *Hint:* Use the connection formula to find the WKB wavefunction inside the classical turning point. Then notice that the infinite potential means that the wavefunction must vanish at $x = 0$.

   (b) Write down the WKB wavefunction for a scattering state $E > 0$. Note that there are two regions where you need to evaluate an integral over $p(x)$; you should evaluate those integrals.

2. **Ionizing an Atom** *from Griffiths 9.18*

   Imagine a hydrogen atom in a small electric field; the electron feels a linear potential from the field, which eventually becomes less than the ground state energy, so it can tunnel out of the atom. In this problem, consider a simple 1D model of this system, with potential

$$V(x) = \begin{cases} \infty\,, & x < -a \\ -V_0\,, & -a < x < 0 \\ -\alpha x\,, & x > 0 \end{cases} . \qquad (2)$$

   (a) Suppose the square well is very deep, so $V_0 \gg \hbar^2/ma^2$. In the absence of the electric field ($\alpha = 0$), what is the approximate ground state energy E? If the electron were a classical particle with this kinetic energy, what would be its speed? *Hint:* You can think of this as the energy of the first odd eigenfunction of a finite square well of width $2a$ or you can approximate the potential as nearly an infinite square well.

   (b) Show that the lifetime of the atom in the presence of the field is $\ln \tau = A|E|^{3/2} + B$, where A and B are constants. Then find A and B (you may need your results from part (a)). *Hint:* the lifetime is the reciprocal of the transition rate defined as in the notes.