PHYS-4602 Homework 8 Due 22 March 2021

This homework is due to `https://uwcloud.uwinnipeg.ca/s/ptx3smosp2xFtmE` by 10:59PM on the due date. You may submit a PDF either scanned from handwriting or generated with LaTeX or a word processor (with an equation editor).

1. **Sharp Kick**

   Consider a particle initially in the ground state of a 1D infinite square well with potential

   $$V(x) = \begin{cases} 0 & 0 < x < a \\ \infty & \text{otherwise} \end{cases} . \tag{1}$$

   At time $t = 0$, the particle receives a kick in the form of a time-dependent potential $\alpha \cos(\pi x/a)\delta(t)$ for small α. What is the probability that the particle is in the first excited state after $t = 0$? (*Hint:* you need to generalize the limits of the time integral slightly for the transition amplitude.)

2. **Exciting a 3D Harmonic Oscillator**

   Consider an electron moving in a 3D harmonic oscillator potential with Hamiltonian

   $$H_0 = \frac{\vec{p}^2}{2m} + \frac{1}{2} m\omega_0^2 \vec{r}^2 . \tag{2}$$

   Starting at time $t = 0$, the electron is exposed to a weak electromagnetic wave moving along z and polarized along x, which introduces a term $H_1(t) = (E_0/\omega) p_x \exp(ikz - i\omega t)$ (plus Hermitian conjugate) to the Hamiltonian. The wavelength is long, so you can approximate that $kz \ll 1$. Recall that the eigenstates of H_0 can be written in terms of x, y, z excitation numbers as $|n_x, n_y, n_z\rangle$ with energies $\hbar\omega_0(n_x + n_y + n_z + 3/2)$.

   (a) Let P_n be the probability that an electron initially in harmonic oscillator state $|n, 0, 0\rangle$ transitions to state $|n + 1, 0, 0\rangle$ at time T. Find the ratio P_n/P_0. You may approximate that the EM wave is spatially uniform.

   (b) In the approximation that the EM wave is spatially uniform, the excitation from $|0, 0, 0\rangle$ to $|1, 0, 1\rangle$ is forbidden (the transition probability vanishes). Using instead the approximation that $\exp(ikz) \sim (1 + ikz)$, find the probability of that transition at time T.

   (c) Consider the transition from $|0, 0, 0\rangle$ to $|1, 0, 1\rangle$ as in the previous part. If transition occurs at long time, what is the frequency of the EM wave?

3. **Variational Calculations**

   (a) Consider a particle moving in 1D in a potential $V(x) = \alpha|x|$. Find the best possible upper bound on the ground state energy using a gaussian trial wavefunction.

   (b) Consider a particle moving in the 1D interval $0 \le x < L$ with periodic boundary conditions on the wavefunction $\psi(0) = \psi(L)$. The particle experiences a potential $V(x)$. Show that the ground state energy of this system is less than or equal to the average value of $V(x)$ over the range $0 \le x < L$.

   (c) *from Griffiths 8.6* Consider a Hamiltonian $H = H_0 + H_1$, where H_0 is exactly solvable and H_1 is small in some sense. Prove that first-order perturbation theory always overestimates the true ground state energy. That is, show that the ground state energy calculated in first-order perturbation theory is greater than (or equal to) the true ground state energy.