PHYS-4602 Homework 6 Due 8 March 2021

This homework is due to `https://uwcloud.uwinnipeg.ca/s/ptx3smosp2xFtmE` by 10:59PM on the due date. You may submit a PDF either scanned from handwriting or generated with LaTeX or a word processor (with an equation editor).

Note: The first three questions are samples taken from last year's mid-term test and are meant to help you see the style of question I tend to ask on tests. The remaining problems are regular homework.

1. **Previous Mid-Term Question #1**

   Is each of the following pairs entangled? Answer yes or no and explain very briefly.

   (a) The spins of an electron and positron in a total spin $s = 0$ state, as we discussed for the EPR experiment.

   (b) Two electrons in an atom in the total angular momentum $|2,0\rangle$ state, which is written as $|2,1\rangle = (|1,1\rangle_1|1,0\rangle_2 + |1,0\rangle_1|1,1\rangle_2)/\sqrt{2}$ in terms of the individual electron orbital angular momenta.

   (c) Two qubits, initially in state $|0\rangle|0\rangle$, after application of the Hadamard gate on each qubit followed by the CNOT gate.

2. **Previous Mid-Term Question #2**

   Suppose someone hands you a qubit and tells you it is 50% likely to be in either of the states $|\pm\rangle = (|0\rangle \pm |1\rangle)/\sqrt{2}$. What is your density operator for this qubit? Show that it is the same as if the other person told you the qubit was 50% likely to be in either state $|0\rangle$ or $|1\rangle$.

3. **Previous Mid-Term Question #3**

   Find the matrix element $\langle n'|(xp + px)|n\rangle$ for a harmonic oscillator. Use your result to write $xp + px$ as a 3×3 matrix for the $n, n' = 0, 1, 2$ states of the oscillator.

4. **Momentum Differentiates the Position Operator**

   Use the rule that $[A, B^n] = n[A, B]B^{n-1}$ when $[A, B]$ commutes with B to prove that $[p, f(x)] = -i\hbar df/dx$, where x and p are 1D position and momentum operators with $[p, x] = -i\hbar$. Assume $f(x)$ can be written as a Taylor series. (Please refer to homework assignment #1 problem 4.)

5. **Gaussian Wavepacket**

   Here we consider the Gaussian wavepacket in 1D at a single instant $t = 0$, ignoring its time evolution. The state is

   $$|\psi\rangle = \int_{-\infty}^{\infty} dx\ Ae^{-ax^2}|x\rangle\ . \tag{1}$$

   Some of these results may be useful on future assignments.

   (a) Find the normalization constant A. *Hint:* To integrate a Gaussian, consider its square. When you square it, change the dummy integration variable to y, then change the integral over $dxdy$ to plane polar coordinates. The textbook cover also has a formula for Gaussian integrals.

   (b) Since the wavefunction is even, $\langle x\rangle = 0$. Find $\langle x^2\rangle$. *Hint:* You can get a factor of x^2 next to the Gaussian by differentiating it with respect to the parameter a.

(c) Write $|\psi\rangle$ in the momentum basis. *Hint:* If you have a quantity $ax^2 + bx$ somewhere, you may find it useful to write it as $a(x + b/2a)^2 - b^2/4a$ by completing the square. Then shift integration variables so it looks like you have a Gaussian again.

(d) Find $\langle p\rangle$ and $\langle p^2\rangle$ and show that this state saturates the Heisenberg uncertainty principle. You should not have to do any integrations.

6. **Harmonic Oscillator Matrix Elements**

   Calculate the matrix elements $\langle n|x|n'\rangle$ and $\langle n|p^2|n'\rangle$ for $|n\rangle, |n'\rangle$ stationary states of the harmonic oscillator. You *must* use Dirac and operator notation and *may not* carry out any integrals.