PHYS-4602 Homework 2 Due 25 Jan 2020

This homework is due to `https://uwcloud.uwinnipeg.ca/s/ptx3smosp2xFtmE` by 10:59PM on the due date. You may submit a PDF either scanned from handwriting or generated with LaTeX or a word processor (with an equation editor).

1. **Measurement vs Time Evolution** *a considerable revision of Griffiths 3.33*

   Suppose a system has observable A with eigenstates $|a_1\rangle, |a_2\rangle$ of eigenvalues a_1, a_2 respectively and Hamiltonian H with eigenstates $|E_1\rangle, |E_2\rangle$ of energies E_1, E_2 respectively. The eigenstates are related by

   $$|a_1\rangle = \frac{1}{5}\left(3|E_1\rangle + 4|E_2\rangle\right) \ , \quad |a_2\rangle = \frac{1}{5}\left(4|E_1\rangle - 3|E_2\rangle\right) \ . \tag{1}$$

   Suppose the system is measured to have value a_1 for A initially. Each of the following parts asks about a different possible set of subsequent measurements.

   (a) What is the probability of measuring energy E_1 immediately after the first measurement? Assuming we do get E_1, what is the probability of measuring a_1 again if we measure A again immediately after the measurement of energy?

   (b) Instead, consider immediately measuring A again after the first measurement. What are the probabilities for observing a_1 and a_2?

   (c) Finally, consider making the first measurement and then allowing the system to evolve for time t. If we then measure energy, what is the probability of finding energy E_1? If we instead measured A again, what is the probability we find a_1 again?

2. **Matrix Hamiltonian**

   Consider a 3D Hilbert space with Hamiltonian

   $$H \simeq E_0 \begin{bmatrix} 0 & 0 & i \\ 0 & 1 & 0 \\ -i & 0 & 0 \end{bmatrix} \tag{2}$$

   in some basis. Work in this basis throughout the problem.

   (a) Show that the time evolution operator is

   $$e^{-iHt/\hbar} \simeq \cos(E_0 t/\hbar) \begin{bmatrix} 1 & 0 & 0 \\ 0 & 1 & 0 \\ 0 & 0 & 1 \end{bmatrix} - i\sin(E_0 t/\hbar) \begin{bmatrix} 0 & 0 & i \\ 0 & 1 & 0 \\ -i & 0 & 0 \end{bmatrix} \tag{3}$$

   in this basis.

   (b) Some operator A is defined in this basis as

   $$A \simeq A_0 \begin{bmatrix} 2 & 0 & 0 \\ 0 & 0 & 0 \\ 0 & 0 & -1 \end{bmatrix} \ . \tag{4}$$

   Suppose the system starts out at time $t = 0$ in a state represented by $[1\ 0\ 0]^T$. Using your previous result, find the state of the system and $\langle A\rangle$ as a function of time. At what times is $\langle A\rangle$ minimized?

(c) Find the uncertainty σ_A of the operator A from the previous part as a function of time.

3. **Kaon Oscillation**

Kaons are subatomic particles; there are two neutral kaons. Like neutrinos, they can oscillate. In the kaon mass eigenbasis $\{|e_1\rangle, |e_2\rangle\}$, the Hamiltonian can be written as the matrix

$$H \simeq \begin{bmatrix} (m+\Delta m)c^2 & 0 \\ 0 & (m-\Delta m)c^2 \end{bmatrix} . \tag{5}$$

The flavor eigenstates are

$$|f_1\rangle = \cos\theta|e_1\rangle + \sin\theta|e_2\rangle \ , \quad |f_2\rangle = -\sin\theta|e_1\rangle + \cos\theta|e_2\rangle \tag{6}$$

for some constant θ.

(a) If a kaon is initially in flavor state $|f_1\rangle$, the probability of measuring it in flavor state $|f_2\rangle$ is a periodic function of time. Find the frequency of that oscillation probability.

(b) What is the uncertainty of the energy for the state $|f_1\rangle$?

(c) Let P_{max} be the maximum oscillation probability. Then we expect one oscillation for every $1/P_{max}$ periods of $P(t)$. Use this length of time as Δt and argue that kaon oscillations respect the "energy-time uncertainty principle" $\Delta E \Delta t \gtrsim \hbar/2$.