- The "Collapse" of the Wavefunction

- EPR Paradox + Bell's Inequality: How real is quantum weirdness?
  - Consider a thought experiment by Einstein, Podolsky, + Rosen
    + Produce 2 qubits in an entangled state. For example, an $e^\pm$ pair (distinguishable particles) in
    $$|s=0\rangle = \left(|\uparrow\rangle_1|\downarrow\rangle_2 - |\downarrow\rangle_1|\uparrow\rangle_2\right)/\sqrt{2}$$
    spin state
    + Send e^- w/A 3 lyr from earth and e^+ w/B 3lyr from earth in the other direction
    + A + B each measure S_z when they receive the same signal (radio, etc) from earth. If A measures $S_z = +\hbar/2$, B must measure $S_z = -\hbar/2$ and vice versa
    + EPR says information can't travel between $e^\pm$ measurements to tell each other how to "collapse wavefunctions"
    + They postulated a hidden variable λ that determines each spin from the beginning (contradicts QM). This is a function $A(\hat{a},\lambda) = \pm 1$ that knows in advance the result A gets from measuring spin in $\hat{a}$ direction $(\hat{a}\cdot\vec{S})$. Same for $B(\hat{b},\lambda)$

  - J. Bell (1964) modified the EPR thought experiment
    + Do the EPR expt except A measures $\hat{a}\cdot\vec{S}_A$ and B measures $\hat{b}\cdot\vec{S}_B$, and they don't know what the other one will measure in advance
    + Do this with many pairs, and then compare results. The QM expectation value is
    $$P(\hat{a},\hat{b}) \equiv 4/\hbar \left\langle (\hat{a}\cdot\vec{S}_A)(\hat{b}\cdot\vec{S}_B)\right\rangle = -\hat{a}\cdot\hat{b}$$
    This is -1 for $\hat{a}=\hat{b}$.

  - Bell proved an inequality for hidden variables
    + From angular momentum conservation — choice of entangled state — the hidden variable functions obey $A(\hat{a},\lambda) = -B(\hat{a},\lambda)$

+ If $p(\lambda)$ is the probability distribution of the hidden variable(s)

$$P(\hat{a},\hat{b}) = \sum_\lambda p(\lambda)\, A(\hat{a},\lambda)\, B(\hat{b},\lambda) \quad \text{could be integral}$$

$$\Rightarrow P(\hat{a},\hat{b}) - P(\hat{a},\hat{c}) = -\sum_\lambda p(\lambda)\, A(\hat{a},\lambda)\left[A(\hat{b},\lambda) - A(\hat{c},\lambda)\right]$$

using the result above.

+ Using triangle inequalities and $|A(\hat{a},\lambda)| = 1$,

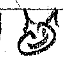
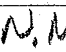
$$\left|P(\hat{a},\hat{b}) - P(\hat{a},\hat{c})\right| \le \sum_\lambda p(\lambda)\left|1 - A(\hat{b},\lambda)A(\hat{c},\lambda)\right|$$

$$= 1 + P(\hat{b},\hat{c}) \qquad \underline{\text{Bell's}}\ \underline{\text{inequality}}$$

- Experimental Results and Meaning
  + The QM prediction violates Bell's inequality when $\hat{a} \perp \hat{b}$ and $\hat{c}$ "in between" — QM cannot be a (local) hidden variables theory
  + Experiments (Aspect et al and subsequent) verify the QM prediction
  + QM is <u>nonlocal</u> in the sense that the wave function "collapses" everywhere at once. But it is local in the sense that it is <u>causal</u> — you can only see the correlation by normal communication after the fact and cannot send messages by measurements. This is fundamentally related to no cloning
  + QM is <u>not</u> <u>"real"</u> — the individual spins/qubits do not have a definite spin along any given direction until they are measured
  + We have to deal with what measurement means

— The Measurement Problem

- The collapse of the wave function is problematic. Consider the unmeasured Schrödinger's Cat

$|\psi\rangle = \frac{1}{\sqrt{2}}\Big(|$  $\rangle|N\rangle + e^{i\phi}\,|$  $\rangle|N, N_e \cdots\rangle\Big)$

+ The usual "wavefunction collapse" is called the <u>Copenhagen Interpretation</u> of QM (after Bohr)

+ But how do we define measurement? Does it require a conscious observer (Wigner, Wheeler)? Can the cat measure itself? Why isn't the observer or measurement device also part of the quantum system?

+ We need a better definition of measurement.

+ In class, we will generally use the Copenhagen interpretation to be concrete adopting the point of view expressed by Mermin: "Shut up and calculate!"

- If we accept that the measurement device + observer are also quantum, measurement is just unitary time evolution of system + observer following the Schr. eqn.
  + The <u>Many Worlds Interpretation</u> (Everett) says the Schr. cat state is actually

$$|\psi\rangle = \tfrac{1}{\sqrt{2}}\left(|\text{cat}\rangle|N\rangle + e^{i\phi}|\text{cat}\rangle|N_1 N_2 \cdots\rangle\right)|\text{obs}\rangle|\text{universe}\rangle$$

  and evolves to

$$|\psi'\rangle = \tfrac{1}{\sqrt{2}}\left(|\text{cat}\rangle|N\rangle\left|{\text{sees} \atop \text{alive}}\right\rangle + e^{i\phi}|\text{cat}\rangle|N_1 N_2 \cdots\rangle\left|{\text{sees} \atop \text{dead}}\right\rangle\right)|\text{universe}\rangle$$

  + The effect is like Copenhagen b/c the observer always sees a definite value, but the wavefunction "branches" as the observer (+ rest of universe) entangle w/ the system. Everything possible happens in some branch.
    These are the "many worlds" ← note that a measurement does not actually create a new universe
  + Everything happens by Schr. eqn. Possibly you can derive the probabilities of different measurements from unitary evolution (Carroll et al)

- Decoherence: roughly the idea that interactions with the environment (a large system) causes the system to evolve toward a definite classical-type eigenstate. (I am combining some slightly different ideas)