PHYS-3203 Homework 9 Due 24 March 2021

This homework is due to `https://uwcloud.uwinnipeg.ca/s/T6ykcP988pa3kpG` by 10:59PM on the due date. You may submit a PDF either scanned from handwriting or generated with LaTeX or a word processor (with an equation editor).

1. **Exploding Cannonball** *inspired by a problem by Barton (and other texts)*

   A cannonball is launched in an arc with velocity $\vec{u}$. At the top of its trajectory, a chemical charge in it explodes into two parts of masses m_1 and m_2 that separate in the horizontal direction only. The explosion releases energy E, which essentially all goes into the kinetic energy of the cannonball pieces. Show that they are separated by a distance $(u_y/g)\sqrt{2E(m_1+m_2)/m_1m_2}$ when they land, where u_y is the initial vertical component of the velocity.

2. **Pucks on an Air Table**

   Imagine an introductory lab experiment in which students collide identical disks moving frictionlessly on a horizontal table. One disk is initially at rest while the other moves toward it at speed v. The disks have mass m and radius a; they each therefore have moment of inertia $I = ma^2/2$ around their own center. All motion is in the plane of the table.

   (a) Argue that the total angular momentum and kinetic energy are given by

   $$\vec{J} = M\vec{R}\times\dot{\vec{R}} + \mu\vec{r}\times\dot{\vec{r}} + I\vec{\omega}_1 + I\vec{\omega}_2\ ,\quad T = \frac{M}{2}\dot{\vec{R}}^2 + \frac{\mu}{2}\dot{\vec{r}}^2 + \frac{1}{2}I\vec{\omega}_1^2 + \frac{1}{2}I\vec{\omega}_2^2\ , \tag{1}$$

   where M is the total mass, μ is the reduced mass, $\vec{R}$ is the overall CM position, $\vec{r}$ is the relative separation, and $\vec{\omega}_{1,2}$ are the angular velocities of the two disks. *Hint:* First change to the overall CM frame, then break the motion of each disk into its own CM motion and motion around its own CM. Note that the formula for angular momentum is similar to one we used but did not derive in our discussion of tidal friction.

   (b) Suppose that the two disks are initially not rotating and move together at relative speed v and impact parameter b. (Recall that the impact parameter is the projection of the relative separation of the centers of the two disks perpendicular to the relative velocity.) After the disks collide, they are no longer sliding along the table (the center of each disk is at rest) but are both rotating with the same angular velocity $\vec{\omega}$. What is the angular speed $\omega \equiv |\vec{\omega}|$? How much kinetic energy is lost in the collision? Based on your answer, what is the largest possible value of b that allows this type of collision?