PHYS-3203 Homework 8 Due 17 March 2021

This homework is due to `https://uwcloud.uwinnipeg.ca/s/T6ykcP988pa3kpG` by 10:59PM on the due date. You may submit a PDF either scanned from handwriting or generated with LaTeX or a word processor (with an equation editor).

1. **Alternating Masses** *from Thornton & Marion*

   Consider a light string loaded with massive beads with uniform spacing l between the beads where the beads at odd positions have mass m and at even positions have mass M. (That is, the first bead from the fixed end has mass m, the next M, and the next m, etc.) There is tension F along the entire string. Assume that both ends of the string are fixed to a wall and that the total number N of beads is odd.

   (a) Study transverse oscillations by guessing a normal mode solution $y_{2j+1} = a_1 \sin[(2j+1)\gamma] \exp(i\omega t)$, $y_{2j} = a_2 \sin(2j\gamma) \exp(i\omega t)$. Then show that the EOM can be written as an eigenvalue problem for the frequency, where $[a_1, a_2]^T$ is the eigenvector.

   (b) Using the boundary conditions, find the allowed values of γ. Solve the eigenvalue problem you found above and show that there are two normal mode frequencies for each allowed value of γ.

   (c) Consider the limit that $m \ll M$. Find the normal mode frequencies to first order in $1/M$ and unnormalized normal mode vectors to lowest nonvanishing order in $1/M$. Show that the lower frequency is the same as the frequency of a loaded string with just the mass M beads and describe the motion of the string for both normal modes.

2. **Vibration of String from Initial Conditions**

   Consider transverse oscillations of a string of linear mass density μ stretched between two supports a distance L apart with tension F. The phase velocity of waves on the string is $v = \sqrt{F/\mu}$. The ends of the string are fixed in place.

   (a) *from Kibble & Berkshire* Suppose the string is plucked so that its initial configuration is two linear segments from ends to the midpoint, which has displacement a (the initial shape of the string is an isosceles triangle of base L and height a). The initial velocity of the string is zero. Describe the subsequent motion of the string and sketch its shape at times $t = L/4v, L/2v, L/v$. Use the general solution to the wave equation, **not** the normal modes/separation of variables.

   (b) Now consider initial conditions such that the string has $y(x,0) = a\sin(3\pi x/L)$ and $\dot{y}(x,0) = 0$. Describe the subsequent motion of the string. *Hint:* if you think about the normal modes of the string, you don't have to do any calculations.

   (c) Now suppose that one end of the string is tied in place, while the other is tied to a hoop that can slide freely along its support, so the boundary conditions are $y(0,t) = 0, y'(L,t) = 0$. What are the normal modes and their frequencies? *Hint:* the wave equation is the same, just the boundary conditions are different.