PHYS-3203 Homework 6 Due 24 Feb 2021

This homework is due to https://uwcloud.uwinnipeg.ca/s/T6ykcP988pa3kpG by 10:59PM on the due date. You may submit a PDF either scanned from handwriting or generated with IAT_EX or a word processor (with an equation editor).

1. Steady Precession of a Top related to question from Thornton & Marion

Consider a symmetric top that is initially spinning vertically $\theta = 0$, so the two conserved angular momenta p_{ϕ} and p_{ψ} as defined in the notes equal each other.

- (a) Write the effective potential $U(\theta)$ for motion in θ and expand it to order θ^2 .
- (b) Based on your expansion above, under what conditions is rotation with $\theta = 0$ stable? Compare this to our discussion of steady precession in the lecture notes. *Hint:* Is $\theta = 0$ an extremum? If so, what kind?
- (c) Suppose that $2MgRI < p_{\psi}^2 < 4MgRI$. By considering the value of $U(\theta)$ at $\theta = 0$ and $\theta = \pi/2$ and the result of the previous part, argue that the top nutates between $\theta = 0$ and another angle $\theta_1 < \pi/2$.
- (d) Find $\dot{\phi}$ and $\dot{\psi}$ when $\theta = 0$. Under what condition does $\dot{\psi}$ have the opposite sign of p_{ψ} for $\theta = 0$? (When this occurs, the top spins in the opposite direction of the precession.)