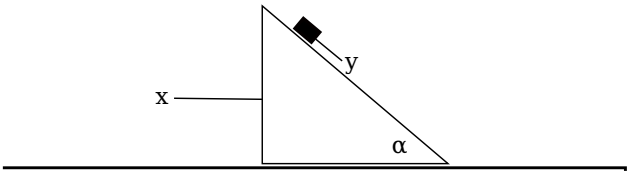
PHYS-3203 Homework 4 Due 3 Feb 2020

This homework is due to `https://uwcloud.uwinnipeg.ca/s/T6ykcP988pa3kpG` by 10:59PM on the due date. You may submit a PDF either scanned from handwriting or generated with LaTeX or a word processor (with an equation editor).

1. **Box on a Wedge**

   A triangular wedge of mass M is able to slide frictionlessly on a horizontal surface, and a box of mass m can slide frictionlessly down the wedge as in the figure below. The incline of the wedge makes an angle α with the horizontal.

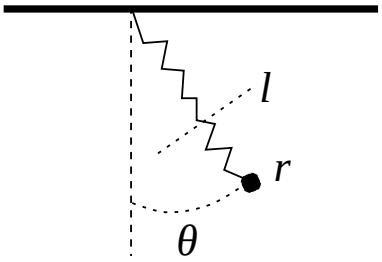


   (a) Write the Lagrangian for this system in terms of x, the displacement of the wedge along the horizontal surface, and y, the displacement of the box down the incline.

   (b) Using the Euler-Lagrange equations, find the acceleration of the wedge as the box slides down the incline (as a multiple of the gravitational acceleration g).

2. **Springy Pendulum**

   A mass m hangs from a spring of negligible mass, which in turn hangs from a pivot which allows motion in a plane under the influence of gravity. The spring and mass therefore form a pendulum that can also oscillate radially. The spring has spring constant k and equilibrium length l. See the figure below.



   (a) Find the Lagrangian for this pendulum in terms of plane polar coordinates as shown in the figure above.

   (b) Find the canonical momenta and Hamiltonian for this system.

   (c) Write Hamilton's equations for this pendulum and find the equilibrium position (where all time derivatives vanish).

   (d) Suppose the pendulum starts at rest at position $r = l$, $\theta = \theta_0$. Describe qualitatively the behavior of r as the pendulum swings down to $\theta = 0$ and explain your answer.