PHYS-3203 Homework 2 Due 20 Jan 2020

This homework is due to `https://uwcloud.uwinnipeg.ca/s/T6ykcP988pa3kpG` by 10:59PM on the due date. You may submit a PDF either scanned from handwriting or generated with LaTeX or a word processor (with an equation editor).

1. **More on the Brachistochrone**

   Consider a particle moving along a brachistochrone path, written parametrically as

   $$x = a\,(\theta - \sin\theta)\ , \quad y = a\,(1 - \cos\theta)\ , \tag{1}$$

   under the influence of gravity (without friction). Note that y increases downward. Show that the time for the particle to move from $x = y = 0$ to the minimum of the curve at $x = \pi a, y = 2a$ can be written as

   $$T = \frac{1}{\sqrt{2g}} \int_0^\pi d\theta \sqrt{\frac{(dx/d\theta)^2 + (dy/d\theta)^2}{y}} \tag{2}$$

   and show that the time for the object to reach the bottom is $T = \pi\sqrt{a/g}$.

2. **A Line Really Is Minimum Length** *from Thornton & Marion and others*

   We know that the minimum length curve in two dimensions that connects the origin to the point $x = y = a$ is the straight line $y(x) = x$. Consider instead the curve $y(x) = x + b\sin(n\pi x/a)$, which also connects the origin to $x = y = a$ if n is an integer. Write the length of this curve as an integral over x and, assuming $b \ll a$, expand the integrand to second order in b/a. From this, show that the length of the curve to this order is $\sqrt{2}a + cb^2/a$, where c is a positive number, and find c. So changing the line to a slightly different curve increases the path length.

3. **Geometry with Constraints** *adapted from problems by Thornton & Marion*

   The following are constrained optimization problems in multivariable calculus. Use the method of Lagrange multipliers to solve them.

   (a) Consider a right-circular cylinder of radius r and height h. If the volume of the cylinder is fixed to V, what is the ratio r/h that minimizes the surface area?

   (b) Consider a parallelepiped circumscribed by a sphere of radius R (that is, the vertices of the parallelepiped lie on the surface of the sphere). What are the dimensions (lengths of the 3 independent sides) of such a parallelepiped of maximum volume?