PHYS-3203 Homework 1 Due 13 Jan 2020

This homework is due to `https://uwcloud.uwinnipeg.ca/s/T6ykcP988pa3kpG` by 10:59PM on the due date. You may submit a PDF either scanned from handwriting or generated with LaTeX or a word processor (with an equation editor).

1. **Crossing the Line**

   A light ray travels through a medium with index of refraction n_1 for $x < 0$ and index n_2 for $x > 0$ starting at position $(-X, 0)$ and ending at position (X, Y) for $X \gg Y$. Use Snell's Law to show that the travel time is minimized when $y = n_2 Y/(n_1 + n_2)$. *Hint:* use the fact that $\tan\theta \approx \sin\theta$ for small angles.

2. **Geodesic on a Cone** *based on a Kibble & Berkshire problem*

   A *geodesic* is the minimal length curve on a surface between two points on that surface (or possibly in a curved space). For example, we showed that a straight line segment is a geodesic on a plane, and you may know that a great circle is a geodesic on a sphere. Here we will examine geodesics on a cone with its tip at the origin and its axis of symmetry along the z axis. The surface of the cone is at a polar angle α from the z axis.

   (a) Find the relationship between the cylindrical coordinates ρ and z on the surface of the cone and show that the distance L from point (ρ_1, φ_1) to point (ρ_2, φ_2) on the cone can be written

   $$L = \int_{\varphi_1}^{\varphi_2} d\varphi \, \sqrt{\rho^2 + \csc^2\alpha \, \rho'^2} \qquad (1)$$

   where $\rho' = d\rho/d\varphi$. *Hint:* you might find the distance formula for cylindical coordinates from Cline appendix C.2.2 useful.

   (b) Show that a geodesic satisfies the equation

   $$\rho\rho'' - 2(\rho')^2 - \sin^2\alpha \, \rho^2 = 0 \ . \qquad (2)$$

   (c) Solve (2) for $\rho(\varphi)$ by changing variables to $\rho = 1/u$. Leave your solution in terms of 2 undetermined integration constants (do not find them in terms of the boundary conditions stated above). What do the integration constants describe?

   (d) As viewed from the origin, a geodesic of infinite length only spans a finite angle $\Delta\varphi$. Find $\Delta\varphi$. Explain the consistency of your answer with what we know about geodesics in the plane (ie, the limit as $\alpha \to \pi/2$).