PHYS-3202 Homework 9 Due 20 Nov 2019

This homework is due in the dropbox outside 2L26 by 10:59PM on the due date. You may alternately email a PDF (typed or black-and-white scanned) or give a hardcopy to Dr. Frey.

1. **Inertia Tensor of a Thin Plate**

   Consider a thin plate of material, which is effectively two-dimensional (this is known as a *lamina*). In this problem, assume that the lamina lies in the $z = 0$ plane.

   (a) *from Taylor* Prove that the moments of inertia $I_{zz} = I_{xx} + I_{yy}$ and the products of inertia $I_{xz} = I_{yz} = 0$.

   In the rest of the problem, assume that the lamina is a rectangle of sides with length $2a$ parallel to the x axis and length a parallel to the y axis. The center of the rectangle is at the origin. The lamina has uniform mass surface density and total mass M. *based on questions from FC and KB*

   (b) Calculate the inertia tensor around this origin given these axes. You may use the results of part (a) to simplify your calculations. (This inertia tensor applies to rotation around any axis through the center of the lamina.) What are the principal axes?

   (c) Use the parallel axis theorem to find the inertia tensor of the lamina around the corner located at $x = -a, y = -a/2$ (so that the x and y axes lie along the sides and z is still perpendicular).

   (d) If the lamina rotates around the long edge through this corner with constant frequency ω, what is the angular momentum as a function of the angle between the lamina and the xy plane? What torque must the axis exert on the lamina as a function of that angle?

2. **Octant of a Sphere** *based on Fowles & Cassiday*

   Consider a solid object of uniform density and mass M in the shape of one octant of a solid sphere of radius a. That is, it consist of all points with $r \leq a$ and $x > 0, y > 0, z > 0$.

   (a) Find the inertia tensor for rotations around the origin. *Hint:* Although the tensor is given in Cartesian coordinates, the integrals can still be carried out in spherical polar coordinates. You can also use symmetry to reduce your calculations.

   (b) Show that a rotation with angular velocity $\vec{\omega} = \omega(\hat{\imath} + \hat{\jmath} + \hat{k})$ is around a principal axis by showing that the angular momentum for this rotation $\vec{J} \propto \vec{\omega}$ and find the principal moment.