PHYS-3202 Homework 8 Due 13 Nov 2019

This homework is due in the dropbox outside 2L26 by 10:59PM on the due date. You may alternately email a PDF (typed or black-and-white scanned) or give a hardcopy to Dr. Frey.

1. **The Airy Rotating Earth**

   In this problem, we will combine the effects of the rotation of earth with air resistance. Use coordinates x, y, z rotating with the earth, where z points up (local vertical), x points east, and y points north. The angular velocity ω of the earth is in the yz plane at an angle θ from the z axis. Assume that any object moving through the atmosphere experiences a quadratic air resistance $\vec{F}_{air} = -mC|\vec{v}|\vec{v}$, where the coefficient C is a property of the specific object and $\vec{v}$ is the velocity of the object with respect to the air. This is a somewhat realistic view of motion on earth.

   (a) Write Newton's 2nd law for the x, y, z coordinates of the object. Assume that the centrifugal force is included in the apparent gravity but make no other approximation. Assume that there is no wind, that is, the air is at rest with respect to the rotating axes.

   (b) These equations must be solved numerically. Code these equations into Maple. Start by defining the constants g, ω, C; use $g = 9.8$ m/s, $\theta = \pi/4$, $\omega = 2\pi/(24 \times 3600)$ 1/s, and $C = 0.01$ 1/m. Then enter the equations of motion as

   ```
   eqx:=diff(x(t),t$2)+···;
   eqy:=diff(y(t),t$2)+···;
   eqz:=diff(z(t),t$2)+···;
   ```

   where $\cdots$ represent the rest of the equations of motion you found above. Set initial conditions appropriate to the object starting at the origin with initial speed v in the xz plane at an angle α from the x axis using the code

   ```
   icx1:=x(0)=0; icx2:=D(x)(0)=v*cos(α); icy1:=y(0)=0; ···
   ```

   and set $v = 45$ m/s and $\alpha = \pi/6$. Then solve for the motion of the object and extract the numerical solutions using Maple code

   ```
   sol := dsolve([eqx, eqy, eqz, icx1, icx2, icy1, icy2, icz1, icz2], numeric,
   output = listprocedure);
   xsol := rhs(sol[2]); ysol := rhs(sol[4]); zsol := rhs(sol[6]);
   ```

   These three functions are the x, y, z coordinates of the object as functions of time. Plot the function `zsol` versus time to see when the object hits the ground again. To get a more specific answer, use the code

   ```
   thit:=fsolve(zsol(t)=0,t=···)
   ```

   where $\cdots$ are a number value near the intercept from your plot. (You may want to check that `zsol(thit)` is small, since the numerical solver `fsolve` seems to be very sensitive to the initial guess.) Finally, find x, y at this time to determine the range x and deflection y of the object. Attach your Maple code.

   (c) The initial conditions are realistic for a baseball that travels a horizontal range of about 130 m. Redo the Maple calculation from the previous part, altering the value of C until you find a value with x range between 120 and 140 m. Based on how the x range changes and the value of the y deflection, is air resistance or the Coriolis force typically more important? *You do not need to attach your Maple code.*

2. **Some Eigenvectors** *inspired by Riley, Hobson, & Bence and Arfken & Weber*

   Define the matrices

$$A = \begin{bmatrix} 0 & 0 & 1 \\ 0 & 1 & 0 \\ 1 & 0 & 0 \end{bmatrix} \quad , \quad B = \begin{bmatrix} 0 & 1 & 1 \\ 1 & 0 & 1 \\ 1 & 1 & 0 \end{bmatrix} \quad , \quad C = \begin{bmatrix} 5 & 0 & 2 \\ 0 & 1 & 0 \\ 2 & 0 & 2 \end{bmatrix} . \tag{1}$$

   (a) By matrix multiplication, show that the vectors $\vec{x} = \begin{bmatrix} 1 & 0 & -1 \end{bmatrix}^T$, $\vec{y} = \begin{bmatrix} 1 & 1 & 1 \end{bmatrix}^T$, and $\vec{z} = \begin{bmatrix} 1 & -2 & 1 \end{bmatrix}^T$ are eigenvectors of both A and B and find their eigenvalues for each matrix.

   (b) Because $\vec{x}, \vec{y}, \vec{z}$ are orthogonal to each other, they form a set of basis vectors, ie, any vector can be written as a linear combination of them. Show that the matrix products $AB = BA$ when multiplying any vector $\vec{v}$.

   (c) Find an orthonormal basis of eigenvectors for C. *Hint:* If you choose carefully, you can find the eigenvectors to be orthogonal from the start. Otherwise, you may need to use the Gram-Schmidt process to orthonormalize your initial choice (you can find the Gram Schmidt process in a mathematical physics textbook if necessary).

3. **Pauli Matrices**

   The Pauli matrices of quantum mechanics are defined by

$$\sigma_x = \begin{bmatrix} 0 & 1 \\ 1 & 0 \end{bmatrix} \quad , \quad \sigma_y = \begin{bmatrix} 0 & -i \\ i & 0 \end{bmatrix} \quad , \quad \sigma_z = \begin{bmatrix} 1 & 0 \\ 0 & -1 \end{bmatrix} \quad . \tag{2}$$

   Find the eigenvalues and eigenvectors for all three Pauli matrices. Normalize the eigenvectors so that their norms are equal to one.