PHYS-3202 Homework 7 Due 6 Nov 2019

This homework is due in the dropbox outside 2L26 by 10:59PM on the due date. You may alternately email a PDF (typed or black-and-white scanned) or give a hardcopy to Dr. Frey.

1. **Puck on Turntable** *inspired by Taylor*

   Consider a puck of mass m on a turntable with angular velocity $\vec{\omega} = \omega \hat{k}$, ie, counterclockwise rotation. Work in a rotating frame with origin at the center of the turntable and x, y axes along set lines on the turntable.

   (a) Write Newton's second law for the x and y coordinates. If we define $u = x + iy$, what is the equation of motion for u? Assume that there is no friction and that ω is constant.

   (b) Solve for $x(t)$ and $y(t)$ given the initial conditions $\vec{r}(0) = x_0 \hat{x}$ and $\dot{\vec{r}}(0) = v_x \hat{x} + v_y \hat{y}$. Sketch your answer for $x_0 = 0, v_y = 0$.

   (c) Explain qualitatively why you might prefer to work in this rotation reference frame as opposed to an inertial frame if the puck and turntable experience kinetic friction opposite $\vec{v}$ or a drag force $-2m\gamma\vec{v}$, where $\vec{v}$ is the relative velocity of the puck and turntable.

   (d) Now consider the case that the puck is initially at rest at $x = r, y = 0$ and $\omega = \dot{\omega}t$. The puck and turntable have coefficient of static friction μ_s. What is the maximum allowed value of $\dot{\omega}$ such that the puck can remain at rest with respect to the turntable at $t = 0$?

2. **Rotating Cylindrical Space Vessel** *inspired by KB 5.9*

   One possible way residents of a space ship or space station can experience "artificial gravity" is if the vessel rotates around some axis. Consider a cylindrical vessel of radius R rotating with angular frequency ω around the cylinder axis.

   (a) The centrifugal force provides an effective gravitational force for stationary objects. At what height h above the "ground" at radius R a second story in a building have to be in order to have only 90% of the gravitational acceleration?

   (b) A train runs around the circumference of the vessel in the direction of the vessel's rotation with speed v relative to the ground. What is the increase in the effective weight (as measured by the normal force) for objects on the train compared to their weight at rest on the ground? Explain your answer first from the rotating frame of the vessel and then from the rotating frame in which the train is at rest. *Hint:* in the frame of the ground, the objects on the train have a centripetal acceleration.

3. **Deflection of Thrown Object** *based on problems from FC and TM*

   (a) Suppose I throw a ball straight upward with initial velocity v. Show that it lands west of me and determine how far in terms of v, the angular velocity of the Earth, and the colatitude. Use the same approximations as in the class notes on the deflection of a falling object.

   (b) Suppose I throw the ball at an angle α from the vertical toward the east. Is the ball deflected north or south? Does it land before or after I expect it would ignoring the Coriolis force?

4. **Satellite and Space Junk** *FC 5.18 clarified and extended*

   A satellite is in a circular orbit of radius R around the Earth. A small bit of space junk of mass m, such as a screw dropped by an astronaut, orbits near the satellite. Use a rotating

coordinate system with origin at the satellite, with $\hat{x}$ pointing away from earth, $\hat{y}$ pointing in the direction of the satellite's velocity, and $\hat{z}$ in the direction of the satellite's angular velocity, to analyze the motion of the space junk. Assume that the space junk stays near the satellite, in other words, that $|\vec{r}| \ll R$, where $\vec{r}$ is the position of the space junk relative to the satellite.

(a) Show that the gravitational force on the space junk is approximately

$$\vec{F}_{grav} = -m\omega^2(\vec{R} + \vec{r}) + 3m\omega^2 x\hat{x} \ , \tag{1}$$

where $\vec{R}$ is the position of the satellite with respect to the Earth. *Hint:* recall that $\omega^2 = GM/R^3$ based either on Kepler's 3rd law or the equality of gravitational and centripetal force on the satellite.

(b) Include the centrifugal and Coriolis forces to show that the equations of motion for the space junk are

$$\ddot{x} = 3\omega^2 x + 2\omega\dot{y} \ , \quad \ddot{y} = -2\omega\dot{x} \ , \quad \ddot{z} = -\omega^2 z \ . \tag{2}$$

(c) Solve (2) and show that z oscillates harmonically about $z = 0$ with frequency ω, x oscillates harmonically about some value x_0 with frequency ω and amplitude x_1, and y oscillates harmonically with amplitude $2x_1$ and frequency ω around a linear trajectory with velocity $v = -3\omega x_0/2$.

(d) Explain qualitatively the sign of the y velocity v in the solution above.