

PHYS-3202 Homework 5 Due 9 Oct 2019

This homework is due in the dropbox outside 2L26 by 10:59PM on the due date. You may alternately email a PDF (typed or black-and-white scanned) or give a hardcopy to Dr. Frey.

1. A Different Central Force *Taylor 8.25 clarified*

Consider a particle of mass m moving in a central force $\vec{F} = -(k/r^{5/2})\hat{r}$, where k is a positive constant. You will need to use Maple to do this problem; attach a printout of your Maple code to your assignment. Label which parts of the code correspond to the different parts of the problem.

- What is the potential energy $V(r)$ for this force?
- Write the effective potential for the particle if the angular momentum is \vec{J} . Find the radius r_0 of a circular orbit. In units where m , k , and J are all 1 (we have to choose mass, length, and time units to do this), use Maple to plot the effective potential over the range $0 < r < 5r_0$. Either use the `view` option or the axis properties menu from right-clicking the plot to set the vertical range to $-0.5 < U < 1$.
- In the units used for your plot in part (b), the particle has energy $E = -0.1$. Use Maple's `fsolve` function to find the minimum and maximum radius that it can reach. Give your answers to 3 significant digits; you may need to set the starting value option for `fsolve` to find both radii.
- Following our discussion in the lecture notes for the inverse square law, write the conserved energy in terms of the variable $u = 1/r$ and its derivative with respect to ϕ . Then take the derivative of this expression with respect to ϕ to find a second order equation for u as a function of ϕ . Leave m, k, J as variables in this part.
- Use the Maple `dsolve` command with the `numeric` option to find $u(\phi)$ for $0 \leq \phi \leq 8\pi$. Use the units from the plot in part (b) and initial conditions $u(0) = 1/r_{min}$, $du/d\phi(0) = 0$, where r_{min} is the minimum radius you found in part (c). Then plot your solution; either use plot options or the axes properties and axes gridlines properties menus from right-clicking the plots to put vertical gridlines at all integer multiples of π and horizontal gridlines as Maple chooses. Does the orbit appear to be closed, ie, is the period of $u(\phi)$ a rational multiple of π to allow u to return to its initial value at $\phi = 2\pi n$ for some integer n ? To do the plot, you may use the `odeplot` command as in the previous assignment or follow the instructions in this guide: http://www.math.unb.ca/~seahra/resources/M4503/maple/dsolve_numeric.pdf .

2. 'Oumuamua *extended from FC 6.17*

The earth's orbit is nearly circular. In this problem, assume that it is exactly circular with radius 1 AU and period 1 year. Note that 1 AU is approximately 1.5×10^8 km and 1 year is approximately 3×10^7 s.

- Find the earth's orbital speed v_{\oplus} in terms of its orbital radius r_{\oplus} and physical constants (Newton's constant, solar mass, earth mass, etc).
- An object is observed at a distance d AU from the sun moving at a speed given by $f v_{\oplus}$. Find the condition on d and f that determines if the object has an elliptical or hyperbolic orbit.

- (c) In October 2017, the PanSTARRS telescope discovered a small object later named 'Oumu-mua (the Hawaiian word for “scout”). On 10 Oct 2017, it was at a distance of 1 AU moving at a speed of about 50 km/s relative to the sun. The perihelion distance was approximately 0.25 AU. Use your result from above to determine if its orbit is elliptical or hyperbolic. If the orbit is elliptical, determine the aphelion distance; if the orbit is hyperbolic, determine the asymptotic speed relative to the sun. Give your answers to 1 significant digit.

3. **Hohmann Transfer** *adapted from KB 4.16 and others*

Suppose we want to send a space probe from earth to Jupiter. The most fuel-efficient orbit for the probe is known as a *Hohmann transfer*. Its perihelion is at earth's orbit (assumed circular), and its aphelion is at Jupiter's perihelion. Jupiter's orbit has semi-major axis $a_J = 5.2$ AU, eccentricity $e_J = 0.049$ and period $T_J = 12$ years. The earth orbit radius is $a_{\oplus} = 1$ AU and period is $T_{\oplus} = 1$ year.

- (a) Find the perihelion distance for Jupiter and the required semi-major axis of the transfer orbit for the spaceprobe. Give your answers first in terms of $a_J, e_J, T_J, a_{\oplus}, T_{\oplus}$ and then in AU.
- (b) What are the eccentricity and period of the transfer orbit? Approximating Jupiter's orbit as a circle, how much angular distance does Jupiter travel while the probe is in transit from earth? Again, give your answers first in terms of the same variables and then as numbers with appropriate units.

4. **Energy Relationships**

- (a) Show that the effective potential for an attractive inverse square force is minimized for $r = \ell$, meaning \dot{r} is maximized there. Then show that $U(\ell) = k/2\ell$.
- (b) Show that the total energy of an elliptical orbit is $E = k/2a$.