

## PHYS-3202 Homework 10 Due 29 Nov 2019

This homework is due in the dropbox outside 2L26 by 10:59PM on the due date. You may alternately email a PDF (typed or black-and-white scanned) or give a hardcopy to Dr. Frey.

### 1. Flipping Box *clarified KB 9.3*

Consider a uniform cubic box of mass  $M$  and side  $2a$  sliding frictionlessly on a surface with velocity  $\vec{v} = v\hat{i}$ . The sides of the cube are oriented in the  $\hat{i}, \hat{j}, \hat{k}$  directions. The cube hits a barrier at  $x = 0$  (extending in  $y$ ), which stops the front edge instantaneously.

- What is the angular velocity of the box around the barrier immediately after the collision?  
*Hint:* use conservation of angular momentum around the barrier.
- Energy is conserved after the collision. What is the minimum initial speed the cube needs to flip all the way over after the collision? Assume that the edge of the cube that first hits the barrier stays at  $x = 0$  afterwards. *Hint:* note that the center of mass must, at a minimum, be directly over the edge if the cube flips over under the given assumption.

### 2. Start Up of Rolling *many places, including FC*

A thin hoop of mass  $M$  and radius  $R$  is spinning around the axis through its center with the axis held horizontally. The initial angular velocity is  $\omega_0$  when the hoop is placed on a surface with coefficient of kinetic friction  $\mu_k$  (at time  $t = 0$ ). When does the hoop stop slipping (ie, begin rolling without slipping)? How far has it traveled since then?

### 3. Wobbling Disc *from FC 9.15*

Consider a disc-shaped symmetric object with  $I_1 = I_2 \equiv I < I_3$ , which is appropriate for a flat cylinder. While spinning in the air, it experiences a drag-like torque  $\vec{\tau} = -k\vec{\omega}$  known as air friction. The disc initially has angular velocity with  $\omega_3 \gg \omega_1, \omega_2$  in terms of the components along the principal axes. *Hint:* this type of torque is one that can be analyzed easily using Euler's equations.

- Show that the angular velocity around the symmetry axis  $\hat{e}_3$  decreases exponentially in time.
- Next, show that the angle between  $\vec{\omega}$  and the symmetry axis decreases in time. That is,  $(\omega_1^2 + \omega_2^2)^{1/2}$  decreases more rapidly in time than  $\omega_3$ .
- Finally, argue that  $\vec{\omega}$  goes to a fixed angle in the  $\hat{e}_1, \hat{e}_2$  plane as  $t \rightarrow \infty$ . You may use your solution to the previous part.