PHYS-4602 Homework 7 Due 12 March 2020

This homework is due in the dropbox outside 2L26 by 10:59PM on the due date. You may alternately email a PDF (typed or black-and-white scanned) or give a hardcopy to Dr. Frey.

1. **Matrix Perturbation Theory**

   Consider the matrix Hamiltonian
   $$H \simeq \begin{bmatrix} E_1 & \epsilon \\ \epsilon & E_2 \end{bmatrix} \tag{1}$$
   with $E_1 \neq E_2$ except when you are told otherwise. Assume that $\epsilon \ll E_1, E_2$.

   (a) To first order in perturbation theory, find the energy eigenvalues and eigenstates.

   (b) What is the first order correction to the energy if $E_1 = E_2 = E$?

   (c) Find the energy eigenvalues to second order in perturbation theory.

   (d) Find the energy eigenvalues and eigenstates exactly. Then expand them as a power series in ϵ and compare to your perturbative answers from parts (a,c). In the case that $E_1 = E_2 = E$, how does your answer compare to part (b)?

2. **Sharp Kick**

   Consider a particle initially in the ground state of a 1D infinite square well with potential
   $$V(x) = \begin{cases} 0 & 0 < x < a \\ \infty & \text{otherwise} \end{cases} . \tag{2}$$
   At time $t = 0$, the particle receives a kick in the form of a time-dependent potential $\alpha \cos(\pi x/a)\delta(t)$ for small α. What is the probability that the particle is in the first excited state after $t = 0$?

3. **Exciting a 3D Harmonic Oscillator**

   Consider an electron moving in a 3D harmonic oscillator potential with Hamiltonian
   $$H_0 = \frac{\vec{p}^2}{2m} + \frac{1}{2} m\omega_0^2 \vec{r}^2 . \tag{3}$$
   Starting at time $t = 0$, the electron is exposed to a weak electromagnetic wave moving along z and polarized along x, which introduces a term $H_1(t) = (E_0/\omega) p_x \exp(ikz - i\omega t)$ (plus complex conjugate) to the Hamiltonian. The wavelength is long, so you can approximate that $kz \ll 1$. Recall that the eigenstates of H_0 can be written in terms of x, y, z excitation numbers as $|n_x, n_y, n_z\rangle$ with energies $\hbar\omega_0(n_x + n_y + n_z - 3/2)$.

   (a) Let P_n be the probability that an electron initially in harmonic oscillator state $|n, 0, 0\rangle$ transitions to state $|n + 1, 0, 0\rangle$ at time T. Find the ratio P_n/P_0. You may approximate that the EM wave is spatially uniform.

   (b) In the approximation that the EM wave is spatially uniform, the excitation from $|0, 0, 0\rangle$ to $|1, 0, 1\rangle$ is forbidden (the transition probability vanishes). Using instead the approximation that $\exp(ikz) \sim (1 + ikz)$, find the probability of that transition at time T.