PHYS-4602 Homework 2 Due 23 Jan 2020

This homework is due in the dropbox outside 2L26 by 10:59PM on the due date. You may alternately email a PDF (typed or black-and-white scanned) or give a hardcopy to Dr. Frey.

1. **The Last Eigenvector**

   A system with a three-dimensional Hilbert space has Hamiltonian represented by the matrix

   $$H \simeq \frac{E_0}{3} \begin{bmatrix} 1/2 & -1/2 & 2i \\ -1/2 & 1/2 & -2i \\ -2i & 2i & -1 \end{bmatrix} . \tag{1}$$

   Two of the eigenstates are represented by the column vectors

   $$|1\rangle \simeq \frac{1}{\sqrt{2}} \begin{bmatrix} 1 \\ 1 \\ 0 \end{bmatrix} \text{ and } |2\rangle \simeq \frac{1}{\sqrt{3}} \begin{bmatrix} i \\ -i \\ 1 \end{bmatrix} . \tag{2}$$

   (a) Find the third eigenstate $|3\rangle$ as a column vector, including proper normalization. Use *only* relations between the eigenvectors, not the action of H on them.

   (b) Find all the eigenvalues of H and write H as a matrix in the $\{|1\rangle, |2\rangle, |3\rangle\}$ basis.

   (c) The operator $|3\rangle\langle 3|$ projects any vector $|\psi\rangle$ onto its component in the $|3\rangle$ direction. Write $|3\rangle\langle 3|$ as a matrix in the original basis.

2. **Expectation and Uncertainty**

   Consider an observable L with three eigenvalues $+1$, 0, and -1 and corresponding eigenstates $|+1\rangle, |0\rangle, |-1\rangle$. We have a system in state

   $$|\psi\rangle = \frac{1}{3}\left(|+1\rangle + 2e^{i\beta}|0\rangle + 2|-1\rangle\right) . \tag{3}$$

   (a) What is the probability of measuring each of the three eigenvalues of L?

   (b) Find the expectation value and uncertainty of a measurement of L.

   (c) Another observable A acts on the L eigenbasis as

   $$A|+1\rangle = \frac{1}{\sqrt{2}}|0\rangle \ , \quad A|0\rangle = \frac{1}{\sqrt{2}}\left(|+1\rangle + |-1\rangle\right) \ , \quad A|-1\rangle = \frac{1}{\sqrt{2}}|0\rangle \ . \tag{4}$$

   Find the expectation value and uncertainty of A in state $|\psi\rangle$.

   (d) Finally, show that the uncertainties of L and A satisfy the uncertainty principle in this state.

3. **Quantum Reality or Not**

   To answer this question, you will need to watch the video of Sidney Coleman's famous lecture "Quantum Mechanics In Your Face" at `http://media.physics.harvard.edu/video/?id=SidneyColeman_QMIYF` or `https://www.youtube.com/watch?v=EtyNMlXN-sw` . (This is about an hour and supplements the reading, which is not long this week.)

(a) The Bell experiment considers 2 distinguishable spin 1/2 particles in the singlet ($s = 0$) total spin state. If $\hat{a}$ and $\hat{b}$ are two unit vectors, show that

$$\left\langle \left(\hat{a} \cdot \vec{S}^{(1)}\right) \left(\hat{b} \cdot \vec{S}^{(2)}\right) \right\rangle = -\frac{\hbar^2}{4} \hat{a} \cdot \hat{b} \, . \tag{5}$$

Hint: Think about a convenient choice of axes and remember that the spin operators are given in matrix form as $S_i \simeq (\hbar/2)\sigma_i$ in terms of the Pauli matrices.

(b) Three electrons are prepared in the so-called "GHZM" spin state $|\psi\rangle = (|+\rangle_1 |+\rangle_2 |+\rangle_3 - |-\rangle_1 |-\rangle_2 |-\rangle_3)/\sqrt{2}$ described in the video. Show that $|\psi\rangle$ is an eigenstate of the operator $S_x^{(1)} S_y^{(2)} S_y^{(3)}$ and find the eigenvalue.